


<p>المادة: الرياضيات الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم - ٢ - المدة : ساعتان</p>	<p>الهيئة الأكاديمية المشتركة قسم : الرياضيات</p>	 <p>المركز العلمي للبحوث والأبحاث</p>
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نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points).

The following statements are true. Justify.

- 1) The complex plane is referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$. Consider the distinct points A, B and C with affixes a, b and c so that $\frac{c-a}{b-a} = 2i$; therefore, A is on the circle with diameter [BC].
- 2) If $\frac{\pi}{2}$ is an argument of z, then $|i+z| = 1 + |z|$.
- 3) If $z = 3\sqrt{3} + 3i$ then z^3 is pure imaginary.
- 4) If $z = e^{i\theta}$, then $z^2 + \frac{1}{z^2}$ is real.
- 5) $|i\bar{z} + 1| = |z + i|$.

II- (4 points)

Given, in the space of an orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, the point A(2 ; 1 ; 5) and the two straight

lines (d) and (d') defined by: (d) $\begin{cases} x = 2m + 4 \\ y = 2m + 1 \\ z = -3m - 5 \end{cases}$ and (d') $\begin{cases} x = t + 2 \\ y = 2t - 1 \\ z = -2t + 6 \end{cases}$, where m and t are real

numbers. Let (P) be the plane determined by A and (d').

- 1) a) Show that A is neither on (d), nor on (d')
b) Prove that (d) and (d') are skew.
- 2) a) Show that $2x + y + 2z - 15 = 0$ is an equation of the plane (P).
b) Prove that (d) is parallel to (P).
- 3) Let (Δ) be the line through A and parallel to (d).
a) Write a system of parametric equations for (Δ).
b) Find the coordinates of B, the meeting point of (Δ) and (d').
- 4) Let E be the orthogonal projection of A on (d').
a) Calculate the coordinates of E.
b) Calculate the area of the triangle AEB.
- 5) Let M be a point on (d), calculate the volume of tetrahedron MABE.

III- (4 points)

U_1 and U_2 are two given urns such that:

U_1 contains 10 balls: 6 red and 4 yellow.

U_2 contains 10 balls: 5 red, 4 black and 1 green.

A fake coin is given such that the probability of having a head is three times more than that of having a tail.

The coin is tossed.

- If we get a tail, we select, randomly and **simultaneously**, two balls from urn U_1
- If we get a head, we select at random two balls from U_2 **one after the other with replacement**.

Consider the following events:

U_1 : "The selected urn is U_1 ."

R: "The selected balls are red."

- 1) Show that $P(U_1) = \frac{1}{4}$.
- 2) Calculate $P(R/U_1)$ and $P(R \cap U_1)$. Deduce that $P(R) = \frac{13}{48}$.
- 3) The two selected balls are red. Calculate the probability that they come from U_1 .
- 4) Let X be the random variable that represents the number of red balls obtained. Determine the probability distribution of X .

IV- (8 points)

Part A


Consider the function g defined over $]1, +\infty[$ as : $g(x) = x (\ln x)^2 - e$.

- 1) Determine the limit of $g(x)$ as $x \rightarrow 1$ and $x \rightarrow +\infty$.
- 2) a) Set up the table of variations of g .
b) if $x > e$, prove that $g(x) > 0$.

Part B

Let f be a function defined over $]1, +\infty[$ as $f(x) = e \frac{\ln x - 1}{\ln x} - x$. Let (C) be the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$ and (d) the line with equation $y = e - x$.

- 1) a) Find $\lim_{x \rightarrow 1} f(x)$; deduce an asymptote to (C).
b) Prove that (d) is an asymptote to (C).
c) Prove that (C) is below (d).
- 2) a) Prove that $f'(x) = \frac{-g(x)}{x \ln^2 x}$.
b) Set up the table of variations of f .
c) Draw (C).
- 3) a) For $1 < x \leq e$, prove that f has an inverse function h . Determine the domain of h .
b) Draw the curve (C') of h in the same system as that of (C).
- 4) Let (Δ) be the line with equation $y = -x - e$.
a) Determine the coordinates of point A, the meeting point of (C') and (Δ).
b) Write an equation of (T), the tangent at A to the curve (C').
c) Solve graphically the inequality $h(x) + e > -x$.

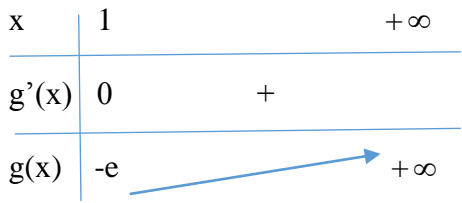
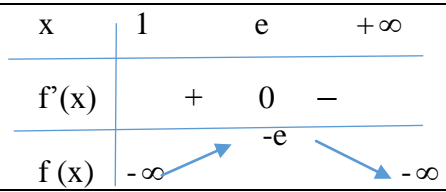
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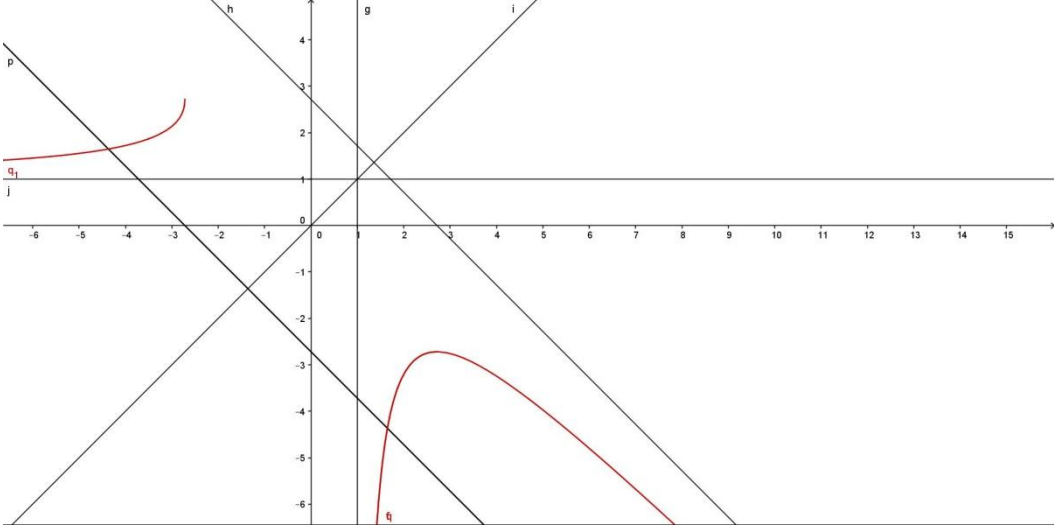
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

Question I (4 points)		Mark
1	$\frac{c-a}{b-a} = 2i$ then $\frac{z_{AC}}{z_{AB}} = \frac{\pi}{2}$ and $(\overline{AB}, \overline{AC}) = \frac{\pi}{2}$. Therefore, A is on the circle with diameter [BC].	0.75
2	$\arg(Z) = \frac{\pi}{2}$ then $z = ib$ with $b > 0$. $i+z = i(1+b)$ and $ i+z = 1+b$. $1+ z = 1+ ib = 1+b$.	0.75
3	$Z = 3\sqrt{3} + 3i = 6e^{i\frac{\pi}{6}}$ and $z^3 = 216e^{i\frac{\pi}{2}}$ pur imaginary.	0.75
4	$Z = e^{i\theta}$ then $z^2 + \frac{1}{z^2} = e^{2i\theta} + e^{-2i\theta} = 2\cos(2\theta)$ (real).	0.75
5	$ i\bar{z} + 1 = i(a-ib) + 1 = 1+b+ia = \sqrt{(1+b)^2 + a^2}$ $ z+i = a+i(b+1) = \sqrt{a^2 + (b+1)^2}$.	1

Question II (4 points)		Mark
1.a	A is not on (d) and A is not on (d').	0.5
1.b	(d) and (d') are neither parallel nor intersecting .	0.5
2.a	A and (d') satisfy the equation: $2x + y + 2z - 15 = 0$ (P).	0.5
2.b	$\vec{V}_d(2,2,-3) \cdot \vec{n}_p(2,1,2) = 0$ and (d) is not on (P) then (d) is parallel to (P).	0.5
3.a	(Δ): $x = 2k+2$, $y = 2k+1$, $z = -3k+5$.	0.5
3.b	B(4,3,2) for $k=1$ and $t=2$.	
4.a	For $E\left(\frac{8}{3}; \frac{1}{3}; \frac{14}{3}\right)$ with $\overline{AE} \cdot \overline{U}_{(d')} = 0$	0.5
4.b	Area = $\frac{AE \cdot EB}{2} = 2u^2$	0.5
5	$V = \frac{1}{6} \overline{AM} \cdot (\overline{AB} \wedge \overline{AE}) = \text{constant}$ because (d) is parallel to (P).	0.5

Question III (4 points)		Mark
1)	$P(U_1) + P(\overline{U_1}) = 1$ and $P(\overline{U_1}) = 3P(U_1)$; then $P(\overline{U_1}) = \frac{3}{4}$ and $P(U_1) = \frac{1}{4}$	0.5
2)	$P(R/U_1) = \frac{1}{3}$ $P(R \cap U_1) = P(R/U_1) \times P(U_1) = \frac{1}{12}$ and $P(R \cap \overline{U_1}) = P(R/\overline{U_1}) \times P(\overline{U_1}) = \frac{3}{16}$ $P(R) = P(R \cap U_1) + P(R \cap \overline{U_1}) = \frac{13}{48}$	0.25 0.5 0.5 0.5
3)	$P(U_1/R) = \frac{4}{13}$	0.5
4)	$X_\Omega = \{0, 1, 2\}$; $P(X=0) = \frac{53}{240}$; $P(X=1) = \frac{61}{120}$; $P(X=2) = \frac{13}{48}$	0.25 0.5 0.5

Question IV (8 points)		Mark
Part A		
1	$x \rightarrow 1, g(x) \rightarrow -e$ $x \rightarrow +\infty, g(x) \rightarrow +\infty$	0.5
2.a	$g'(x) = (\ln x)^2 + 2x(\ln x) \frac{1}{x} = \ln x(\ln x + 2)$ 	0.5
2.b	$g(e)=0$, then $g(x) > 0$ for $x > e$.	0.5
Part B		
1.a	$x \rightarrow 1, f(x) \rightarrow -\infty$ then $x=1$ is a vertical asymptote. $x \rightarrow +\infty, f(x) \rightarrow -\infty$.	0.25
1.b	$\lim_{x \rightarrow +\infty} [f(x) - e + x] = \lim_{x \rightarrow +\infty} e \left[\frac{\ln x - 1}{\ln x} - 1 \right] = 0$ Then (d) is an asymptote to (C).	0.5
1.c	$f(x) - y = e \left(\frac{-1}{\ln x} \right) < 0$ then (C) is below (d).	0.5
2.a	$f'(x) = e \left(\frac{1}{x \ln^2 x} \right) - x = \frac{-g(x)}{x \ln^2 x}$.	0.75
2.b		0.5

2.c		1
3.a	For $x \in]1;e]$, f is continuous and strictly increasing, then f has an inverse function h . $Dh=]-\infty,-e]$.	0.5
3.b	Plot (C')	0.5
4.a	<p>(Δ) is the symmetric of (Δ) with respect to $y=x$. Find $(\Delta) \cap (C)$:</p> $e\left(\frac{\ln x - 1}{\ln x}\right) - x = -x - e.$ $\ln x = \frac{1}{2} \text{ and } x = \sqrt{e}$ <p>$(\Delta) \cap (C) = A'(\sqrt{e}, -\sqrt{e} - e).$</p> <p>$(\Delta) \cap (C') = A(-\sqrt{e} - e, \sqrt{e})$</p>	0.5
4.b	$f'(x) = \frac{-g(x)}{x \ln^2 x} ; f'(\sqrt{e}) = 1 - 4\sqrt{e}.$ <p>Slope of $(T) = \frac{1}{1 - 4\sqrt{e}}.$</p> $(T) : y - \sqrt{e} = \frac{1}{1 - 4\sqrt{e}}(x + \sqrt{e} + e).$	1
4.c	$h(x) > -x - e$. Consider the part of (C') that is above (Δ) then $x \in]-\sqrt{e} - e, -e]$.	0.5