وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة 2001 العادية

عدد المسـائل : أربعة
مسابقة في الرياضيات
المدة: ساعتان
الاسم:
الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات .
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الإلتزام بترتيب المسائل الواردة في المسابقة).

I - (2,5 points)

In the table below, only one of the proposed answers to each question is correct.
Write the number of each question and the corresponding answer.

Nº	Given	Questions	Answers a	Answers b	Answers c	Answers d
1	$z = e^{i\frac{\pi}{6}}$	$z^9 =$	9	$9i$	i	$-i$
2	$z' = \dfrac{z-1}{\bar{z}-1}$ $(z \neq 1)$	$\|z'\| =$	$\|z\|$	$2\|z\|$	1	2
3	$z = \sin\dfrac{\pi}{3} - i\cos\dfrac{\pi}{3}$	$z =$	$e^{i\frac{\pi}{6}}$	$e^{-i\frac{\pi}{6}}$	$e^{i\frac{\pi}{3}}$	$e^{-i\frac{\pi}{3}}$
4	$f(x) = \dfrac{e^x}{x^e}$	$\lim\limits_{x\to+\infty} f(x) =$	1	$+\infty$	0	$\dfrac{1}{e}$
5	$I = \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^3 x + 1)\,dx$	$I =$	π	0	1	$\dfrac{\pi}{2}$

II - (5,5 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the plane (P) of equation $x - 2y + z + 1 = 0$, and the points $A(2; -2; -1)$, $B(1; 0; -2)$ and $C(2; 1; -1)$.

1) Determine an equation of the plane (Q) containing A, B and C.
2) Prove that the planes (P) and (Q) intersect along the straight line (BC).
3) a - Prove that (P) and (Q) are perpendicular.
   b - Calculate the distance from A to (BC).
4) Let (d) be the straight line defined by:

$$\begin{cases} x = t-1 \\ y = t+1 \\ z = t+2 \end{cases}$$ where t is a real parameter.

a- Verify that (d) is included in (P).
b- Let M be a variable point on (d). Prove that the area of triangle MBC is independent of the position of M on (d).

III- (4 points)

In a certain town, 40 % of men are smokers.
It is known that 6 % of men in this town have lung disease. Of those men having lung disease, 85 % are smokers.
A man is selected, at random, from this town.
Consider the following events::

D: « the selected man has a lung disease »
S : « the selected man is a smoker »

1) Determine the following probabilities: p (D) , p (S) and p(S /D).
2) Calculate the probability of each of the following events:
   a- The selected man is a smoker and has a lung disease.
   b- The selected man is a nonsmoker and has a lung disease.
   c- The selected man has a lung disease knowing that he is a smoker.

IV – (8 points)

Consider the functions f and g , defined on $]0\,;+\infty[$ by :

$f(x) = 2x \ln x$ and $g(x) = e^{\frac{1}{2x}}$

Designate by (C) the representative curve of f and by (G) that of g , in an orthonormal system $(O;\vec{i},\vec{j})$. (*unit: 2 cm*).

1) a - Calculate the limits of f at 0 and at $+\infty$ and specify f(e).
   b - Set up the table of variations of f and draw (C).
2) Calculate the area of the domain bounded by the curve (C), the axis of abscissas and the lines of equations $x = 1$ and $x = \sqrt{e}$
3) a - Calculate the limits of g at 0 and at $+\infty$.
   Specify the asymptotes of (G).
   b - Set up the table of variations of g and draw (G) (*in the same system as* (C)).
4) a - Prove that the function g admits, on$]0\,;+\infty[$,an inverse function g^{-1}
   b - Specify the domain of definition of g^{-1} and determine $g^{-1}(x)$ in terms of x.
5) The line of equation $y = 1$ cuts (C) at a point A of abscissa a , and the line of equation $y = x$ cuts (G) at a point B of abscissa b.
   Prove that $a = b$ and verify that $1.4 < a < 1.5$

Sciences de la vie **MATHEMATIQUES** **1ère session**
2001

Questions		ELEMENTS DE REPONSE	Note
I	1	d	½
	2	c	½
	3	b	½
	4	b	½
	5	a	½
II	1	$\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC}) = 0$ (Q) : $x - z - 3 = 0$.	
	2	(P) et (Q) sont distincts. Les coordonnées de B et C vérifient l'équation de (P).	
	3	a) $\overrightarrow{N}(1 ; -2 ; 1)$ est normal à (P) $\overrightarrow{N'}$ (1 ; 0 ; –1) est normal à (Q) et $\overrightarrow{N}.\overrightarrow{N'} = 0$ b) $d_{A/(BC)} = d_{A/(P)} = \frac{\lvert 2+4-1+1\rvert}{\sqrt{1+4+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$.	½ 1
	4	a) $t - 1 - 2t - 2 + t + 2 + 1 = 0$ alors $0 = 0$ b) $\overrightarrow{BC}(1 ; 1 ; 1)$ est le vecteur directeur de (d) alors (d) est parallèle à (BC) et la distance de M à (BC) est constante . Ou calculons la distance de M à (BC) et démontrons qu'elle est constante. Ou calculons l'aire du triangle MBC : $\frac{1}{2}\left\Vert\overrightarrow{MB}\wedge\overrightarrow{BC}\right\Vert = \frac{1}{2}\sqrt{54}$ = constante .	½ 1 ½
III	1	$P(M) = \frac{6}{100} = 0{,}06$. $P(F) = \frac{40}{100} = 0{,}4$. $P(F/M) = \frac{85}{100} = 0{,}85$.	¼ ¼ ½
	2	a) $P(F\cap M) = P(M) . P(F/M) = (0{,}06) . (0{,}85) = 0{,}051$. b) $P(\overline{F}\cap M) = P(M) - P(F\cap M) = 0{,}06 - 0{,}051 = 0{,}009$. c) $P(M/F) = \frac{P(M\cap F)}{P(F)} = \frac{0{,}051}{0{,}4} = 0{,}1275$.	1 1 1

IV

1-a (¼ ¼ ¼ ¼)

$\lim_{x \to 0} f(x) = 0$; $\lim_{x \to +\infty} f(x) = +\infty$

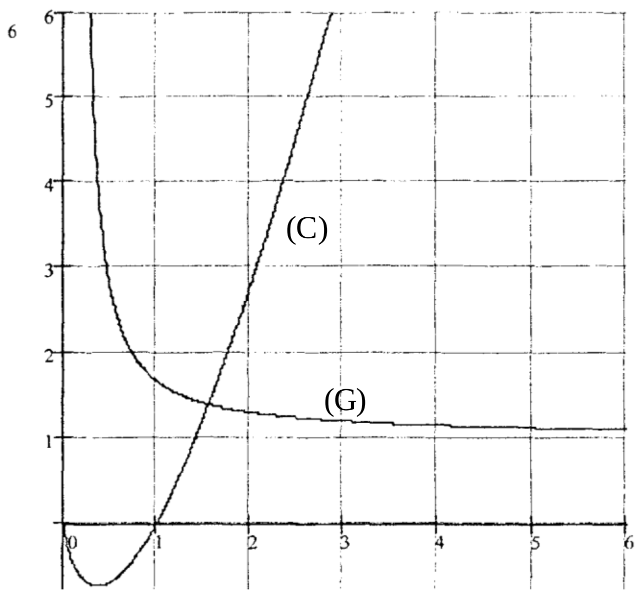
$f(e) = 2e$

$f'(x) = 2(1 + \ln x)$.

1-b (¼ ¼)

x	0		$\frac{1}{e}$		$+\infty$
f'(x)		−	0	+	
f(x)	0	↘	$-\frac{2}{e}$	↗	$+\infty$

(½)



2 (1)

$A = \int_1^{\sqrt{e}} 2x \ln x \, dx = \left[x^2 \ln x - \frac{x^2}{2} \right]_1^{\sqrt{e}} = \frac{1}{2}$ unité de surface $= \frac{1}{2}$ (4 cm^2) = 2 cm^2 .

3-a (¼ ¼ ¼ ¼)

$\lim_{x \to 0} g(x) = +\infty$ et $\lim_{x \to +\infty} g(x) = 1$;

asymptotes : y'y et la droite d'équation x = 1

3-b (¼ ¼)

$g'(x) = \frac{-1}{2x^2} e^{\frac{1}{2x}} < 0$.

x	0		$+\infty$
g'(x)		−	
g(x)	$+\infty$	↘	1

IV

	4	a) g est continue et strictement décroissante, alors elle admet une fonction inverse g^{-1} . b) $D_{g^{-1}} =]1;+\infty[$ $y = e^{\frac{1}{2x}}$; $\ln y = \frac{1}{2x}$; $x = \frac{1}{2\ln y}$ et $g^{-1}(x) = \frac{1}{2\ln x}$.	½ ¼ ½
	5	D'après (C) , l'équation $2x \ln x = 1$ admet une seule racine . L'abscisse a de A est une solution de l'équation alors $2a \ln a = 1$. L'abscisse b de B est une solution de l'équation $e^{\frac{1}{2b}} = b$ Donc on a $\frac{1}{2b} = \ln b$ soit $2b\ln b = 1$, d'où a = b f(1,4) = 0,942 et f(1,5) = 1,216 ; f est continue et f(a) = 1 donc $1,4 < a < 1,5$.	¾

Life sciences **Mathematics** **1st session**
2001

Questions		SHORT ANSWERS	Marks
I	1	d	½
	2	c	½
	3	b	½
	4	b	½
	5	a	½
II	1	$\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC}) = 0$ (Q) : $x - z - 3 = 0$.	1
	2	(P) and (Q) are distinct. The coordonates of B and C verify the equation of (P) .	¼ ¾
	3	a) $\vec{N}(1 ; -2 ; 1)$ is normal to (P) $\vec{N'}(1 ; 0 ; -1)$ is normal to (Q) and $\vec{N} . \vec{N'} = 0$ b) $d_{A/(BC)} = d_{A/(P)} = \frac{\lvert 2+4-1+1\rvert}{\sqrt{1+4+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$.	½ 1
	4	a) $t - 1 - 2t - 2 + t + 2 + 1 = 0$ then $0 = 0$ b) $\overrightarrow{BC}(1 ; 1 ; 1)$ is a direction vector of (d) then (d) is parallel to (BC) and the distance from M to (BC) is constant. Or we calculate the distance from M to (BC) and show that it is constant. Or we calculate the area of triangle MBC as : $\frac{1}{2}\left\lVert \overrightarrow{MB}\wedge\overrightarrow{BC} \right\rVert = \frac{1}{2}\sqrt{54}$ = constant .	½ 1 ½
III	1	$P(M) = \frac{6}{100} = 0.06$. $P(F) = \frac{40}{100} = 0.4$. $P(F/M) = \frac{85}{100} = 0.85$.	¼ ¼ ½
	2	a) $P(F\cap M) = P(M) . P(F/M) = (0.06) . (0.85) = 0.051$. b) $P(\overline{F}\cap M) = P(M) - P(F\cap M) = 0.06 - 0.051 = 0.009$. c) $P(M/F) = \frac{P(M\cap F)}{P(F)} = \frac{0.051}{0.4} = 0.1275$.	1 1 1

<table>
<tr><td rowspan="5">IV</td><td>1-a

1-b</td><td>$\lim_{x\to 0} f(x) = 0$; $\lim_{x\to +\infty} f(x) = +\infty$
$f(e) = 2e$
$f'(x) = 2(1 + \ln x)$.

<table>
<tr><td>x</td><td>0</td><td></td><td>$\frac{1}{e}$</td><td></td><td>$+\infty$</td></tr>
<tr><td>f'(x)</td><td></td><td>–</td><td>0</td><td>+</td><td></td></tr>
<tr><td>f(x)</td><td>0</td><td>↘</td><td>$-\frac{2}{e}$</td><td>↗</td><td>$+\infty$</td></tr>
</table>

(for (C) see 3-b)</td><td>¼ ¼
¼
¼

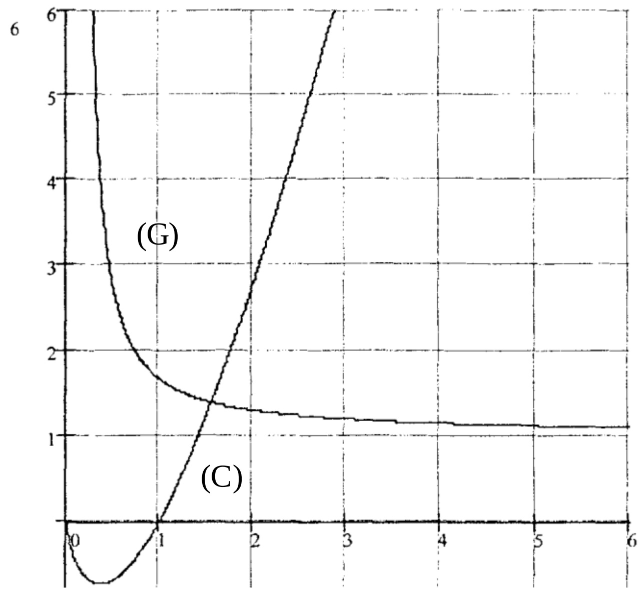
¼

¼</td></tr>
<tr><td>2</td><td>$A = \int_1^{\sqrt{e}} 2x\ln x\,dx = \left[x^2\ln x - \frac{x^2}{2}\right]_1^{\sqrt{e}} = \frac{1}{2}$ units of area = $\frac{1}{2}$ (4 cm^2) = 2 cm^2 .</td><td>1</td></tr>
<tr><td>3-a

3-b</td><td>$\lim_{x\to 0} g(x) = +\infty$ and $\lim_{x\to +\infty} g(x) = 1$; asymptotes : y'y and x = 1

$g'(x) = \frac{-1}{2x^2} e^{\frac{1}{2x}} < 0$

<table>
<tr><td>x</td><td>0</td><td></td><td>$+\infty$</td></tr>
<tr><td>g'(x)</td><td></td><td>–</td><td></td></tr>
<tr><td>g(x)</td><td>$+\infty$</td><td>↘</td><td>1</td></tr>
</table>
</td><td>¼ ¼
¼ ¼
¼

¼</td></tr>
<tr><td>3-b</td><td>
</td><td>½</td></tr>
</table>

IV	4	a) g is continuous and strictly decreasing, then it admits an inverse function g^{-1}. b) $D_{g^{-1}} =]1;+\infty[$ $y = e^{\frac{1}{2x}}$; $\ln y = \frac{1}{2x}$; $x = \frac{1}{2\ln y}$ and $g^{-1}(x) = \frac{1}{2\ln x}$.	½ ¼ ½
	5	From (C) , the equation $2x \ln x = 1$ admits a unique root . The abscissa a of A is the solution of the equation then $2a \ln a = 1$. The abscissa b of B is the solution of the equation $e^{\frac{1}{2b}} = b$ Which can be written $\frac{1}{2b} = \ln b$ then $2b\ln b = 1$, $a = b$. $f(1.4) = 0.942$ and $f(1.5) = 1.216$; f is continuous and $f(a) = 1$ then $1.4 < a < 1.5$.	¾