Life Sciences **MATHEMATIQUES** **2nd session 2001**

Questions		ANSWERS	Mark
I	1	The normals to (P) and (Q) are $\overrightarrow{N_P}(1\,;-3\,;-2)$ $\overrightarrow{N_Q}(3\,;-1\,;-2)$ $\overrightarrow{N_P}$ and $\overrightarrow{N_Q}$ not collinear because $\frac{1}{3}\neq\frac{-3}{-1}$. (P) and (Q) are intersecting along the straight line (D) of equation : Let $x = m$ $\begin{cases}3y+2z=m+6\\-y-2z=-3m-2\end{cases}$ the solution $\begin{cases}y=-m+2\\z=2m\end{cases}$ (D) : $x = m$ $y = -m+2$ $z = 2m$.	**1**
	2	(P) $\cap$ (Q) = (D), A intersection between (D) and (R) then A is common to the three planes (P), (Q) and (R). $m - m + 2 - 2m + 2 = 0$ then $m = 2$ and $A(2\,;0\,;4)$.	**1**
	3	I midpoint of [OA] then I (1 ; 0 ; 2) A symmetric of O with respect to (S) , (S) passes through I and admits as normal $\overrightarrow{OA}(2\,;0\,;4)$ M (x ; y ; z) point of (S) then $\overrightarrow{OA}.\overrightarrow{IM} = 0$ (S) : $2(x-1) + 0(y-0) + 4(z-2) = 0$ (S) : $x + 2z - 5 = 0$ $d(A,S) = \frac{\lvert 2+8-5\rvert}{\sqrt{1+4}}=\frac{5}{\sqrt{5}}=\sqrt{5}$ unit of length.	**1**
II	1	$z' = \frac{z}{\lvert z\rvert^2}$ for $z = 2$ $\lvert z\rvert = 2$ $z' = \frac{2}{4}=\frac{1}{2}$; for $z = \sqrt{2}\,e^{i\frac{\pi}{4}}$ $\lvert z\rvert=\left\lvert\sqrt{2}\,e^{i\frac{\pi}{4}}\right\rvert$ $z = \sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})=\sqrt{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = 1 + i$ $z' = \frac{1+i}{2} = \frac{1}{2}+\frac{i}{2}$; for $z = \sqrt{2}\,e^{-i\frac{\pi}{4}}$ same calculation $z' = \frac{1}{2}-\frac{i}{2}$.	¾
	2	$\lvert z^2\rvert=z.\bar{z}$ therefore $z' = \frac{z}{z\bar{z}}=\frac{1}{\bar{z}}$.	½
	3	$\left(\overrightarrow{OM},\overrightarrow{OM'}\right) = \arg(z') - \arg(z) = \arg(\frac{1}{\bar{z}}) - \arg(z) = 0 \bmod 2\pi$ The three points O , M and M' are collinear.	¾
	4	$z' = \frac{1}{\bar{z}}$; $\overline{z'}=\frac{1}{z}$; $\overline{z'-1}=\overline{z'}-1=\frac{1}{z}-1=\frac{1-z}{z}$	**1**
	5	$z' = \frac{1}{\bar{z}}$ then $\lvert z'\rvert=\left\lvert\frac{1}{\bar{z}}\right\rvert=\frac{1}{\lvert z\rvert}$ because ($\lvert\bar{z}\rvert=\lvert z\rvert$).	½

	6-a	M describes the circle (C) with center E and radius 1 $\lvert z_M - z_E \rvert = 1$; $\lvert z-1 \rvert = 1$; $\lvert 1-z \rvert = 1$. Ou $\lvert 1-z \rvert^2 = \lvert 1-x-iy \rvert^2 = (1-x)^2 + y^2 = 1$.	¾
	6-b	$\lvert z'-1 \rvert = \frac{\lvert 1-z \rvert}{\lvert z \rvert} = \lvert z' \rvert$ $\lvert z-1 \rvert = 1$; $\lVert \overrightarrow{OM'} - \overrightarrow{OE} \rVert = \lVert \overrightarrow{EM'} \rVert$; and $\lvert z' \rvert = OM'$ then M'O = M'E	**1 ¼**
	6-c	M' describes the perpendicular bisector (d) of [OE]. M is a point of (C) M' is a point of (d). The three points O , M and M' are collinear M' is the intersection point of (OM) and (d).	½

III

1

Salary	**Frequency**	**Center of classes (x_i)**	**$n_i x_i$**	$\lvert x_i - \bar{x} \rvert$	**$n_i (x_i - \bar{x})^2$**
[400 ; 600[	8	500	4000	400	1 280 000
[600 ; 800[	5	700	3500	200	200 000
[800 ; 1000[	9	900	8100	0	0
[1000 ; 1200[	15	1100	16500	200	600 000
[1200 ; 1400[	3	1300	3900	400	480 000
Total	**40**		**36000**		**2 560 000**

$\bar{X} = \frac{\sum n_i x_i}{40} = 900$

The average of monthly salaries of these 40 workers is 900 000 LL.

Grade: **1**

2

The variance : $V = \frac{1}{n}\left(\sum n_i (x_i - \bar{x})^2\right) = 64\,009$.

The standard deviation : $\sigma = \sqrt{v} = \sqrt{64009} = 253$.

Grade: ¾

3

13 workers have a salary < 800 000 LL.

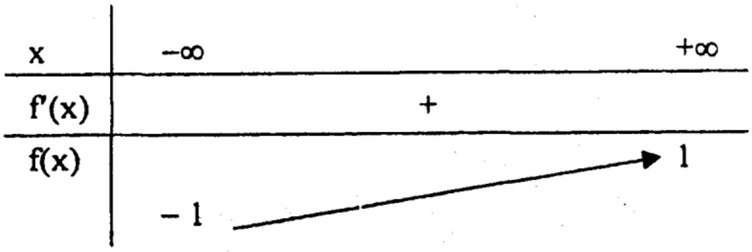
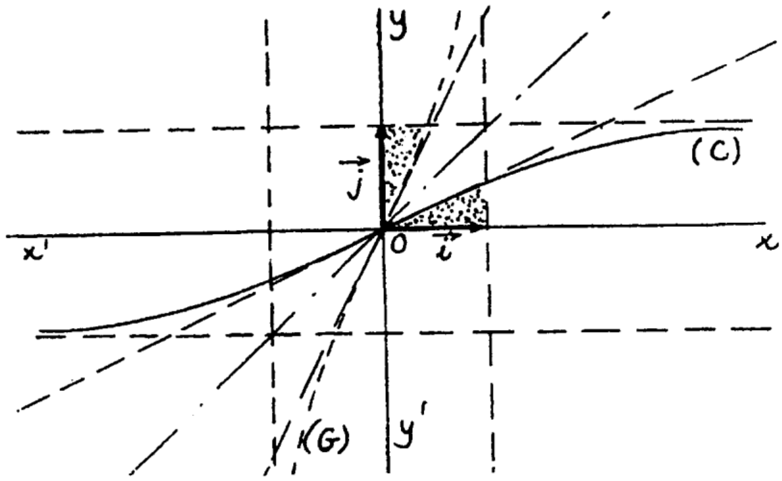
The probability to choose 2 from them is $p = \frac{C_{13}^2}{C_{40}^2} = \frac{13 \times 12}{40 \times 39} = \frac{1}{10} = 0.1$

Remark : the parts 1 and 2 can be found by using a calculator (using the center of classes and the frequency in mode SD).

Grade: **1 ¼**

IV

IV	1	$f(x) + f(-x) = \frac{2e^x}{e^x+1} - 1 + \frac{2e^{-x}}{e^{-x}+1} - 1 = 2 - 2 = 0$. f(x) is defined over $\mathbb{R}$ that centered at O and f(x) + f(–x) = 0 then f(x) is an odd function and (C) admits the origin O as a center of symmetry.	**1**
	2	$\lim_{x \to +\infty} f(x) = 2 - 1 = 1$; y = 1 is an asymptote to (C) at $+\infty$ $\lim_{x \to -\infty} f(x) = -1$; y = – 1 is an asymptote to (C) at $-\infty$	**1**

IV	3	$f'(x) = \frac{2e^x}{\left(e^x+1\right)^2}$; $f'(x) > 0$ over its domain.  	**1**
	4	$f(0) = \frac{2(1)}{1+1} - 1 = 0$; $f'(0) = \frac{1}{2}$ (D) is tangent at O to (C) : $y - 0 = \frac{1}{2}(x - 0)$ D : $y = \frac{1}{2}x$.  	**1 ½**
	5	$A = \int_0^1 f(x)\,dx = \int_0^1 \frac{2e^x}{e^x+1}dx - \int_0^1 dx$ = (2 ln(e + 1) – 2 ln2 – 1) unit of area = 4(2 ln(e + 1) – 2 ln2 – 1) cm^2 . (unit of area = 4 cm^2)	**1**
	6-a	Domain of f^{-1} inverse function of f is]– 1 ; 1[$y = \frac{2e^x}{e^x+1} - 1$; $e^x = \frac{y+1}{1-y}$; $\ln e^x = \ln \frac{y+1}{1-y}$; $x = \ln \frac{y+1}{1-y}$; $f^{-1}(y) = \ln\frac{y+1}{1-y}$	**1**
	6-b	The curve (G) is the symmetric of (C) with respect to y = x	**½**
	6-c	B is the symmetric of A with respect to y = x then area of B = area of A.	**1**