

عدد المسائل : اربع	مسابقة في الرياضيات	الاسم: الرقم:
ة: ساعتان		

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.  
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

### I- ( 3 points)

In the table given below, only one among the proposed answers to each question is correct.

Write down the number of each question and give, **with justification**, the answer corresponding to it.

N°	Questions	Answers			
		a	b	c	d
1	A particular solution of the differential equation : $y'' - 4y' - 5y = 0$ , is :	$y = e^x$	$y = xe^{-x}$	$y = e^{-x}$	$y = \cos 5x$
2	A(1 ; 2 ; -1 ) and B(3 ; 0 , 1) are two given points. An equation of the mediator plane of [AB] is : $x + my + nz - 1 = 0$ where	$m = -1$ $n = 1$	$m = -1$ $n = -1$	$m = 1$ $n = 1$	$m = 1$ $n = -1$
3	<i>(d) is the line of intersection of the two planes :</i> (P) : $x + y - z + 1 = 0$ (Q) : $2x - y + z = 0$ . A direction vector of (d) is :	$\vec{V}(0 ; -1 ; 1)$	$\vec{V}(2 ; -3 ; -3)$	$\vec{V}(0 ; 1 ; 1)$	$\vec{V}(2 ; 3 ; -3)$

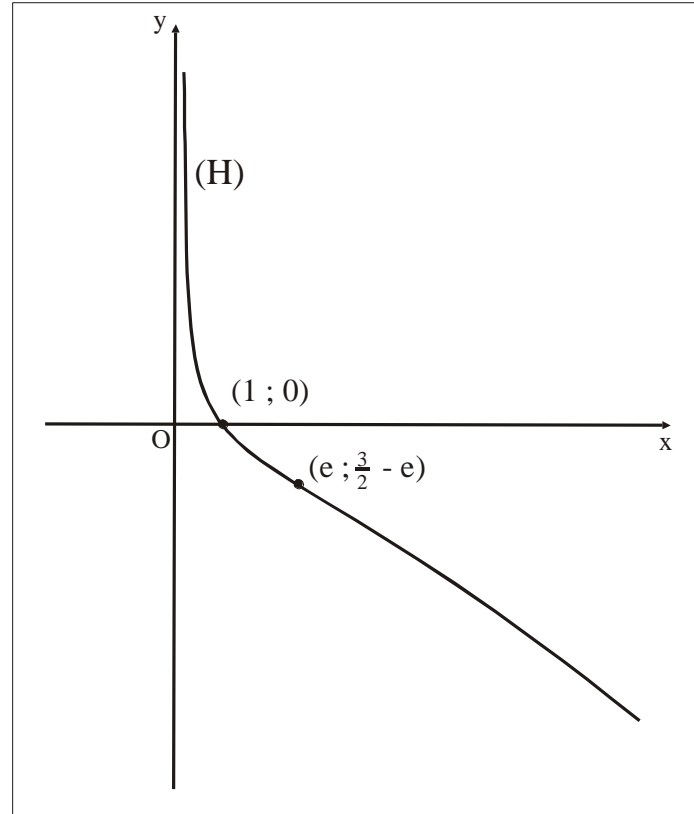
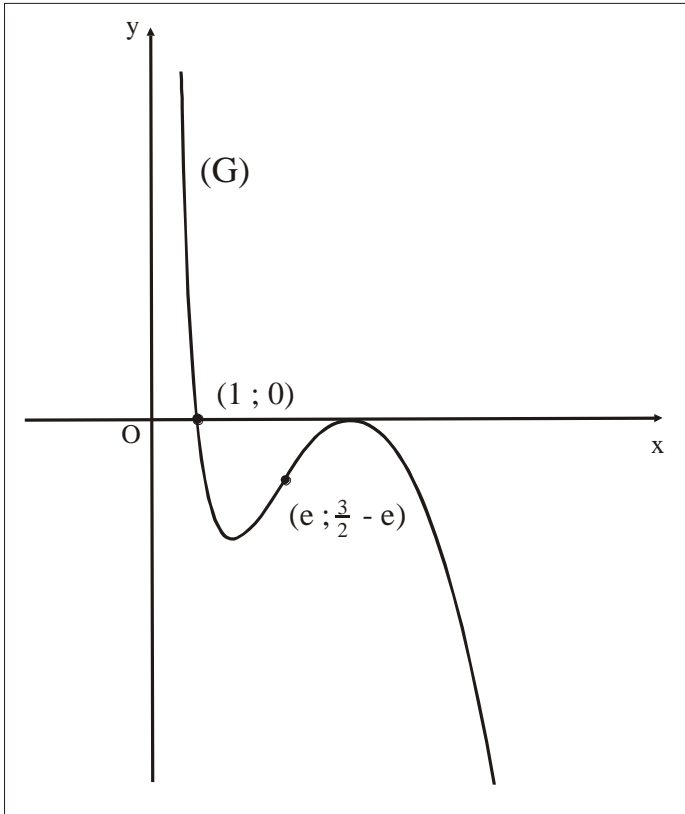
### II- ( 7 points)

Let  $f$  be the function defined, on the interval  $I = ]0 ; + \infty [$ , by :  $f(x) = \frac{\ln x}{x} - 1$ .

Designate by (C) its representative curve in an orthonormal system  $(O ; \vec{i}, \vec{j})$ .  
(unit : 2cm).

- 1) a- Calculate the limits of  $f$  at the boundaries of  $I$ .  
b- Determine the asymptotes of (C).
- 2) Calculate  $f'(x)$  and set up the table of variations of  $f$ .
- 3) Verify that  $y = x - 2$  is an equation of the straight line (d), tangent to (C) at the point A(1 ; -1).

- 4) Plot the line (d) and the curve (C) .  
 5) One of the two curves (G) and (H), shown in the figure below, represents an **antiderivative** (primitive)  $F$  of the given function  $f$  .



- a- Among these two curves, which one represents the function  $F$  ?  
 Justify your answer.
- b- Without finding the expression of  $F(x)$  , calculate in  $\text{cm}^2$  the area of the domain bounded by the curve (C) of  $f$  , the axis of abscissas and the two lines of equations  $x = 1$  and  $x = e$  . Give the answer to the nearest  $10^{-2}$  .

### III- (5 points)

In the complex plane referred to a direct orthonormal system  $(O; \vec{u}, \vec{v})$  , consider the points  $A$  and  $A'$  of affixes  $-4$  and  $4$  respectively.

$M$  being a point in the plane, of affix  $z$  , ( $M$  is distinct from  $A$ ), consider the point

$M'$  of affix  $z'$  such that  $z' = \frac{z - 4}{z + 4}$  .

- 1) Write  $z'$  in its algebraic form in the case when  $z = 4e^{i\frac{\pi}{3}}$  .
- 2) Let  $z = x + iy$  and  $z' = x' + iy'$  . Express  $x'$  and  $y'$  in terms of  $x$  and  $y$  .

*In all what follows, designate by (C) the circle of center O and radius 4, and suppose that M describes the circle (C) deprived of A and A'.*

3) Prove that  $z'$  is pure imaginary.

4) Let  $z = 4 e^{i\theta}$ , where  $0 < \theta < \frac{\pi}{6}$ .

Let N be the point of affix  $\bar{z}$ , and L be the point of affix  $z_1 = 4 e^{i3\theta}$ .

a- Plot the points M, N and L in the preceding system.

b- Verify that  $z_1 = \frac{z^2}{\bar{z}}$ .

c- Prove that MLN is an isosceles triangle of principal vertex M.

#### IV-(5 points)

A family  $F_1$  has **4** children : **two** girls and **two** boys ; a second family  $F_2$  has **3** children : **one** girl and **two** boys.

1) A club management decides to choose, at random, a group of 3 children from these 7 children to spend a free summer vacation abroad.

a- What is the number of possible groups that can be formed ?

b- Prove that the probability of having exactly one girl among the chosen 3 children is  $\frac{18}{35}$ .

c- Let X be the random variable which denotes the number of girls among the chosen 3 children.

Determine the probability distribution (probability law) of X.

2) Due to financial reasons, the club management decides to choose **only one child**.

**To do this, the management chooses a family at random, from which a child is chosen randomly, (the choices are supposed to be equiprobable).**

Consider the event A : « the chosen child is a girl of the family  $F_2$  » .

a- Verify that the probability of A is  $P(A) = \frac{1}{6}$ .

b- Calculate the probability of each of the following events :

B : « the chosen child is a girl of the family  $F_1$  » ,

C : « the chosen child is a girl » ,

D : « the chosen child is from the family  $F_1$  knowing that this child is a girl » .