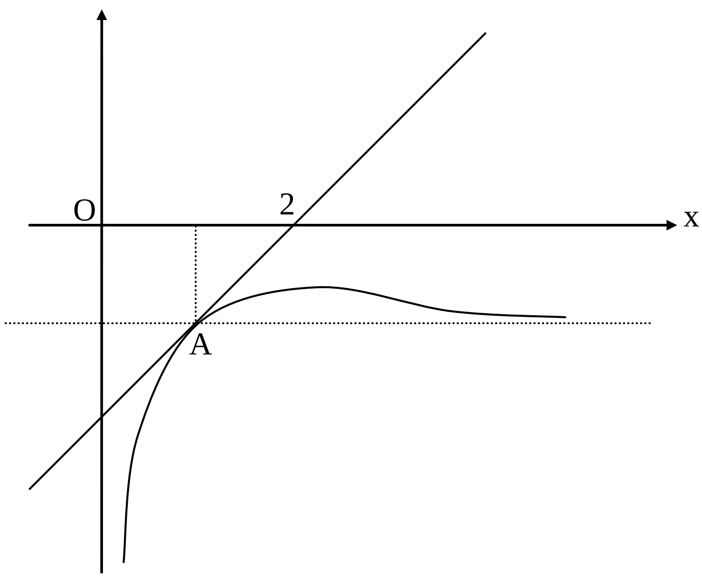
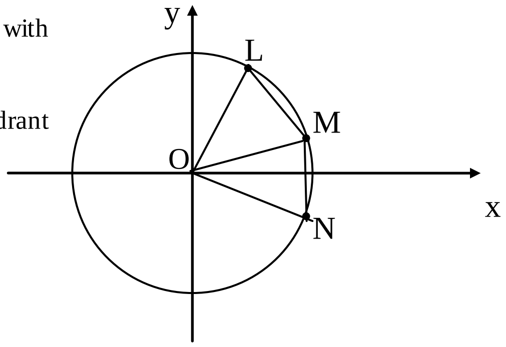
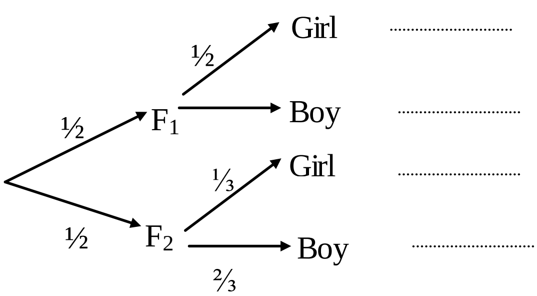
**Life sciences** | **MATHEMATICS** | **1st session 2002**

| Questions | | ELEMENTS OF SOLUTIONS & MARKING SCHEME | |
|---|---|---|---|
| I | 1 | (c): ► By verification<br>► Or $r^2 - 4r - 5 = 0$ , $y = C_1 e^{-x} + C_2 e^{5x}$ ( on prend $C_1 = 1$ and $C_2 = 0$) | **1** |
| | 2 | (a): ► By finding the equation of the plane : $x - y + z - 1 = 0$ .<br>► Or : By noticing that $\overrightarrow{AB}(2\,;-2\,;2)$ is collinear to $\vec{s}(1;m;n)$ which gives : m = - 1 and n = 1 . | **1** |
| | 3 | (c) : ► By finding a system of parametric equations of (d) :<br>$x = -\frac{1}{3}$ , $y = t - \frac{2}{3}$ , $z = t$ .<br>► Or : By verifying that $\vec{V}$ is orthogonal to the two planes. | **1** |
| II | 1(a) | $\lim\limits_{x\to 0^+} f(x) = -\infty$ ; $\lim\limits_{x\to+\infty} f(x) = -1$ | **1** |
| | 1(b) | The straight line of equation x = 0 is a vertical asymptote<br>The straight line of equation y=-1 is a horizontal asymptote | ½ |
| | 2 | $f'(x) = \frac{1-\ln x}{x^2}$<br>$\begin{array}{c\|ccccc} x & 0 & & e & & +\infty \\ \hline f'(x) & & + & 0 & - & \\ \hline f(x) & -\infty & \nearrow & \frac{1}{e}-1 & \searrow & -1 \end{array}$<br>$\frac{1}{e} - 1 \approx -0.65$ | **1** |
| | 3 | $y + 1 = f'(1)(x - 1)$ ; $y = x - 2$ . | **1** |
| | 4 |  | **1 ½** |
| | 5(a) | f(x) < 0 on ] 0 , + ∞[ Hence F is strictly decreasing and then (H) is the representative curve of F . | **1** |
| | 5(b) | The area of the domain is A = \|F(e) – F(1)\| = $(e - \frac{3}{2})u^2 = (4e - 6)\ cm^2 = 4.87\ cm^2$ . | |
| III | 1 | $z = 4e^{i\frac{\pi}{3}}$ , hence $z' = \frac{2+2i\sqrt{3}-4}{2+2i\sqrt{3}+4} = i\frac{\sqrt{3}}{3}$ | **1** |
| | 2 | $z' = x' + iy' = \frac{x-4+iy}{x+4+iy} = \frac{x^2+y^2-16}{(x+4)^2+y^2} + i\frac{8y}{(x+4)^2+y^2}$ | **1** |
| | 3 | M describes the circle (C) ,deprived of A and A’ , then OM = \|z\| =4 and $y \neq 0$ ,<br>Then $x^2 + y^2 = 16$ i. e Re(z’) = 0 and Im(z’) $\neq$ 0 .<br>z’is pure imaginary . | **1** |

<table>
<tr><td rowspan="2"></td><td>4(a)</td><td>N is the symmetric of M with respect to the x-axis<br><br>L belongs to the 1st quadrant<br></td><td>½</td></tr>
<tr><td>4(b)</td><td>$\dfrac{z^2}{\bar{z}} = \dfrac{16e^{2i\theta}}{4e^{-i\theta}} = 4e^{3i\theta} = z_1$</td><td>½</td></tr>
<tr><td rowspan="6">IV</td><td>4(c)</td><td>► $\left(\overrightarrow{ON}, \overrightarrow{OM}\right) = 2\theta \ (2\pi)$ and $\left(\overrightarrow{OM}, \overrightarrow{OL}\right) = 2\theta \ (2\pi)$. Then MN=ML<br>► or MN=$|\bar{z} - z|$ , ML=$|z_1 - z|$ = $\left|\dfrac{z^2}{\bar{z}} - z\right| = \left|\dfrac{z^2 - z\bar{z}}{\bar{z}}\right| = \dfrac{|z|}{|\bar{z}|}|z - \bar{z}| = |z - \bar{z}|$</td><td>1</td></tr>
<tr><td>1(a)</td><td>Each group is a combination of 3 children chosen among the 7 children .<br>The number of groups is ${}_7C_3 = 35$</td><td>½</td></tr>
<tr><td>1(b)</td><td>P(having exactly one girl ) = $\dfrac{C_3^1 \times C_4^2}{35} = \dfrac{18}{35}$</td><td>½</td></tr>
<tr><td>1(c)</td><td><table><tr><td>$X=x_i$</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$p_i$</td><td>4/35</td><td>18/35</td><td>12/35</td><td>1/35</td></tr></table></td><td>1 ½</td></tr>
<tr><td>2(a)</td><td>P(choosing $F_2$ ) = $\dfrac{1}{2}$ , p(A) = $\dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$</td><td>½</td></tr>
<tr><td>2(b)</td><td>► P(B) = $\dfrac{1}{2} \times \dfrac{2}{4} = \dfrac{1}{4}$ ( ½ )<br><br>P(C) = P(A) + P(B) = $\dfrac{5}{12}$ (1)<br><br>P(D) = $\dfrac{P(B)}{P(C)} = \dfrac{3}{5}$ ( ½ )<br><br>► Or :<br><br>P(B) = $\dfrac{1}{4}$<br>P(A) = $\dfrac{1}{6}$</td><td>2</td></tr>
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