وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٣ العادية

| عدد المسائل: اربع | مسابقة في الرياضيات<br>المدة: ساعتان | الاسم:<br>الرقم: |
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ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة) .

**I – (3 points)**

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points E , F and G of respective affixes :

$$z_E = i \quad , \quad z_F = 2 \quad \text{and} \quad z_G = (1+\frac{\sqrt{3}}{2}) + i(\frac{1}{2}+\sqrt{3}).$$

1) Write the complex number $z' = \dfrac{z_G - z_E}{z_F - z_E}$ in its algebraic form, and verify that $z' = e^{i\frac{\pi}{3}}$.

2) Prove that EFG is an equilateral triangle.

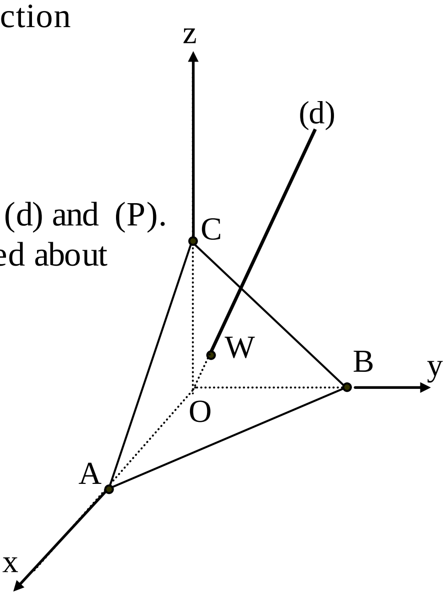
3) Let M be a variable point of affix $z$. Determine the set (T) of points M such that $|z - z_E| = \sqrt{5}$, and verify that F belongs to (T).

**II – (4 points)**

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the plane (P) of equation $x + y + z - 2 = 0$.

1) Determine the coordinates of A, B and C, the points of intersection of the plane (P) with the axes of coordinates.

2) Write a system of parametric equations of the straight line (d) passing through O and perpendicular to the plane (P).




3) a -Determine the coordinates of W, the point of intersection of (d) and (P).
b-Prove that the point W is the center of the circle circumscribed about the triangle ABC.

4) Consider the point $E(\frac{4}{3};-\frac{2}{3};\frac{4}{3})$.

a-Verify that E is the symmetric of B with respect to W.
b-Calculate the area of the quadrilateral ABCE.

**III – (5 points)**

A man has in his right-hand pocket **<u>one</u> bill of 100 000 LL** and **<u>three</u> bills of 20 000 LL**, and has in his left-hand pocket **<u>three</u> bills of 50 000 LL** and **<u>two</u> bills of 20 000 LL .**

1) He draws at random **<u>one bill</u>** from each of these two pockets.
Let X be the random variable that designates the sum of money thus drawn.

a - Prove that $P(X = 70\,000) = \dfrac{9}{20}$.

b - Determine the probability distribution of X.

c - Calculate $P(X < 100\,000)$.

2) In this part, this man chooses one of these two pockets and then he draws, from the chosen pocket, simultaneously **two bills** at random.

Consider the following events:

R : He chooses the right-hand pocket.
L : He chooses the left-hand pocket.
S : The sum drawn is less than 90 000 LL .

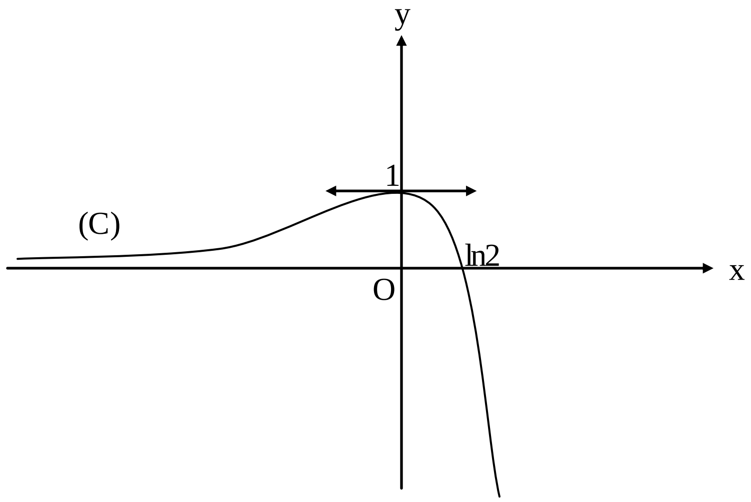
Suppose that $P(R) = \frac{2}{3}$ and $P(L) = \frac{1}{3}$.

a- Calculate the probability P(S/R).

b- Calculate the probability of each of the following events: $S \cap R$ , $S \cap L$ and S .

**IV– (8 points)**

The curve (C) shown in the figure below is the representative curve, in an orthonormal system $(O ; \vec{i} , \vec{j})$, of a function f defined over IR .




1) a -Prove that f admits, over $[0 ; +\infty[$, an inverse function g.
b-Specify the domain of definition of g , and draw its representative curve (G).

2) The function f, represented by the curve (C) in the figure above, is a particular solution of the differential equation $y'' - 3y' + 2y = 0$.
Determine f(x).

**In all what follows, let** $f(x) = 2e^{x} - e^{2x}$ .

3) Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = \ln 2$.

4) Verify that $g(x) = \ln(1 + \sqrt{1-x})$.

5) Calculate g' (x), and deduce the slope of the tangent to the curve (C) at the point A of abscissa ln2.

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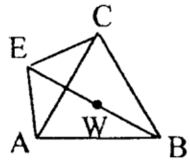
| Questions | | Elements de Réponses | Notes |
|---|---|---|---|
| I - | 1 | $z' = \dfrac{\left(1+\dfrac{\sqrt{3}}{2}\right)+i\left(\dfrac{1}{2}+\sqrt{3}\right)-i}{2-i} = \dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$<br>$e^{i\frac{\pi}{3}} = \cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3} = z'$ | 1 |
| | 2 | EF = $\lvert z_F - z_E \rvert = \lvert 2-i \rvert = \sqrt{5}$ et FG = GE = $\sqrt{5}$<br>► ou $\lvert z' \rvert = 1 = \dfrac{EG}{EF}$ et arg z' = $(\overrightarrow{EF};\overrightarrow{EG}) = \dfrac{\pi}{3}$ donc EFG est équilatéral. | 1 |
| | 3 | $\lvert z - z_E \rvert = \sqrt{5}$ alors EM = $\sqrt{5}$, donc l'ensemble des points M est le cercle (T) de centre E et de rayon $\sqrt{5}$.<br>EF = $\sqrt{5}$ , F appartient à (T).<br>► ou on pose z = x + iy, d'où $\lvert x+iy-i \rvert = \sqrt{5}$ ce qui donne une équation de (T) : $x^2 + (y-1)^2 = 5$ | 1 |
| II- | 1 | A(2 ; 0 ; 0) , B(0 ; 2 ; 0) et c(0 ; 0 ; 2) | ½ |
| | 2 | $\overrightarrow{N_P}$ (1 ; 1 ; 1) est un vecteur directeur de (d) et (d) passe par O.<br>(d) : x = m , y = m , z = m . | ½ |
| | 3a | W(m ; m ; m) avec W est un point de (P), d'où m + m + m = 2 , m = $\dfrac{2}{3}$<br>Par suite W $\left(\dfrac{2}{3};\dfrac{2}{3};\dfrac{2}{3}\right)$ . | ½ |
| | 3b | WA = WB = WC = $\dfrac{2\sqrt{6}}{3}$ , donc W est le centre du cercle circonscrit au triangle ABC.<br>► ou OA = OB = OC avec W est le projeté orthogonal de O sur (ABC),<br>Donc WA = WB = WC | 1 |
| | 4a | On vérifie que W est le milieu de [BE] . | ½ |
| | 4b | L'aire de ABCE est le double de celle du triangle EAB ;<br>S(EAB) = $\dfrac{1}{2}\lvert \overrightarrow{AB} \wedge \overrightarrow{AE} \rvert = \dfrac{4\sqrt{3}}{3}$ $u^2$ .<br>L'aire de ABCE est $\dfrac{8\sqrt{3}}{3}$ $u^2$ .<br><br>► ou aire (ABCE) = $\dfrac{1}{2}$ BE × AC . | 1 |
| III- | 1a | P(X = 70 000) = (tirer 50 000 de p.g) × P(tirer 20 000 de p.d)<br>P(X = 70 000) = $\dfrac{3}{5}\times\dfrac{3}{4} = \dfrac{9}{20}$ | ½ |

<table>
<tr><td rowspan="5">III-</td><td>1b</td><td>
<table>
<tr><td>$x_i$</td><td>40 000</td><td>70 000</td><td>120 000</td><td>150 000</td></tr>
<tr><td>$P_i$</td><td>$\frac{2}{5}\times\frac{3}{4}=\frac{3}{10}$</td><td>$\frac{9}{20}$</td><td>$\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$</td><td>$\frac{3}{5}\times\frac{3}{4}=\frac{3}{10}$</td></tr>
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<tr><td>1c</td><td>$P(X < 100\,000) = \frac{6}{20}+\frac{9}{20}=\frac{15}{20}=\frac{3}{4}$.</td><td>½</td></tr>
<tr><td>2a</td><td>$P(S/D) = \frac{C_3^2}{C_4^2} = \frac{3}{6} = \frac{1}{2}$</td><td>2</td></tr>
<tr><td>2b</td><td>$P(S\cap D)=P(D)\times P(S/D)=\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$<br>$P(S/D) = \frac{C_2^2 + C_2^1\times C_3^1}{C_5^2}=\frac{1+6}{10}=\frac{7}{10}$<br>$P(S\cap D)=\frac{1}{3}\times\frac{7}{10}=\frac{7}{30}$ .<br>$P(S) = P(S\cap D) + P(S\cap G) = \frac{1}{3}+\frac{7}{30}=\frac{17}{30}$</td><td>2</td></tr>
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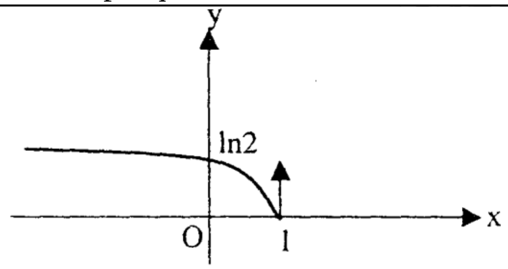
<tr><td rowspan="6">IV-</td><td>1a</td><td>D’après sa courbe représentative, la fonction f est continue et strictement décroissante. Donc elle admet une fonction réciproque.</td><td>½</td></tr>
<tr><td>1b</td><td>$D_g = ]-\infty\ ;\ 1]$ , (G) est symétrique de (C) où $x \geq 0$ par rapport à la droite d’équation $y = x$ .<br>
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<tr><td>2</td><td>L’équation caractéristique associé à l’équation différentielle est :<br>$r^2 - 3r + 2 = 0$ , qui a pour solution $r_1 = 1$ , $r_2 = 2$ d’où : $y = C_1e^x + C_2e^{2x}$<br>on a $y(0) = 1$ et $y(\ln 2) = 0$ (ou $y(0) = 1$ et $y'(0) = 0$), ce qui donne $C_1 = 2$ et $C_2 = -1$ . d’où $f(x) = 2e^x - e^{2x}$</td><td>2</td></tr>
<tr><td>3</td><td>$A = \int_0^{\ln 2}(2e^x - e^{2x})\,dx = \left(2e^x - \frac{1}{2}e^{2x}\right)\Big|_0^{\ln 2} = 2e^{\ln 2} - \frac{1}{2}e^{2\ln 2} - 2 + \frac{1}{2} = 0{,}5u^2$</td><td>1½</td></tr>
<tr><td>4</td><td>$y = 2e^x - e^{2x}$ , donne $2e^x - e^{2x} + y = 0$ , d’où $(e^x - 1)^2 = 1 - y$<br>$x \geq 0$ donne $e^x \geq 1$ et par suite $e^x - 1 = \sqrt{1-y}$ , donc $x = \ln(1+\sqrt{1-y})$<br>la fonction réciproque est alors $g(x) = \ln(1+\sqrt{1-x})$<br>► ou on démontre que $g^{-1}(x) = f(x)$<br>►ou on démontre $(g\circ f)(x) = x$ ou $(f\circ g)(x) = x$</td><td>1</td></tr>
<tr><td>5</td><td>On a $g'(x) = \frac{-1}{2\sqrt{1-x}\,(1+\sqrt{1-x})}$<br>Le point A(ln2 , 0) de (C) a pour symétrique B(0 , ln2 ) de (G).</td><td>1</td></tr>
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| | | La pente de la tangente en A à (C) est $f'(\ln 2) = \dfrac{1}{g'(0)} = -4$ | |

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| Questions | | ANSWERS | scheme |
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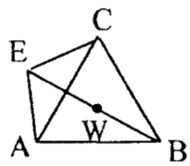
| I - | 1 | $z' = \dfrac{\left(1+\dfrac{\sqrt{3}}{2}\right)+i\left(\dfrac{1}{2}+\sqrt{3}\right)-i}{2-i} = \dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$<br>$e^{i\frac{\pi}{3}} = \cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3} = z'$ | 1 |
| | 2 | EF = $\lvert z_F - z_E\rvert = \lvert 2-i\rvert = \sqrt{5}$ and FG = GE = $\sqrt{5}$<br>► or $\lvert z'\rvert = 1 = \dfrac{EG}{EF}$ and arg z' = $(\overrightarrow{EF};\overrightarrow{EG}) = \dfrac{\pi}{3}$ hence EFG is equilateral. | 1 |
| | 3 | $\lvert z - z_E\rvert = \sqrt{5}$ then EM = $\sqrt{5}$, thus the set of points M is the circle (T) of center E and radius $\sqrt{5}$.<br>EF = $\sqrt{5}$ , F belongs to (T).<br>► or let z = x + iy, then $\lvert x+iy-i\rvert = \sqrt{5}$ which gives an equation of (T) :<br>$x^2 + (y-1)^2 = 5$ | 1 |
| II- | 1 | A(2 ; 0 ; 0) , B(0 ; 2 ; 0) and c(0 ; 0 ; 2) | ½ |
| | 2 | $\overrightarrow{N_P}$ (1 ; 1 ; 1) is a director vector of (d) and (d) passes through O.<br>(d) : x = m , y = m , z = m . | ½ |
| | 3a | W(m ; m ; m) with W is a point (P), hence m + m + m = 2 , m = $\dfrac{2}{3}$<br>Thus W $(\dfrac{2}{3};\dfrac{2}{3};\dfrac{2}{3})$ . | ½ |
| | 3b | WA = WB = WC = $\dfrac{2\sqrt{6}}{3}$ , then W is the center of the circle circumscribed about the triangle ABC.<br>► or OA = OB = OC and W is the orthogonal projection of O on (ABC),<br>So WA = WB = WC | 1 |
| | 4a | We verify that W is the mid point of [BE] . | ½ |
| | 4b | The area of ABCE is double of the area of triangle EAB ;<br>S(EAB) = $\dfrac{1}{2}\lvert\overrightarrow{AB}\wedge\overrightarrow{AE}\rvert = \dfrac{4\sqrt{3}}{3}\ u^2$ .<br>Area ABCE = $\dfrac{8\sqrt{3}}{3}\ u^2$ .<br><br>► Or area (ABCE) = $\dfrac{1}{2}$ BE × AC . | 1 |
| III- | 1a | P(X = 70 000) = (drawing 50 000 from L.P) × P(drawing 20 000 from R.P) | ½ |

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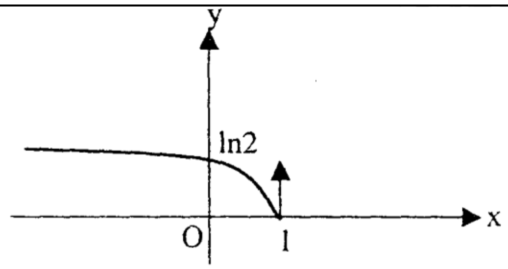
<tr><td rowspan="5">IV-</td><td>1a</td><td>Using the representative curve, we can conform that the function f is continuous and strictly decreasing. Therefore it has an inverse function..</td><td>½</td></tr>
<tr><td>1b</td><td>$D_g = ]-\infty\ ; 1]$ , (G) is symmetric of (C) for $x \geq 0$ with respect to the line of equation $y = x$ .<br>
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<tr><td>2</td><td>The characteristic equation associated to the differential equation is:<br>$r^2 - 3r + 2 = 0$ , which has as solution $r_1 = 1$ , $r_2 = 2$ we get : $y = C_1e^x + C_2e^{2x}$<br>we have $y(0) = 1$ and $y(\ln 2) = 0$ (or $y(0) = 1$ and $y'(0) = 0$), consequently<br>$C_1 = 2$ and $C_2 = -1$ . Hence $f(x) = 2e^x - e^{2x}$</td><td>2</td></tr>
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<tr><td>4</td><td>$y = 2e^x - e^{2x}$ , is equivalent to $2e^x - e^{2x} + y = 0$ , $(e^x - 1)^2 = 1 - y$<br>$x \geq 0$ leads to $e^x \geq 1$ then $e^x - 1 = \sqrt{1-y}$ , and $x = \ln(1+\sqrt{1-y})$<br>$g(x) = \ln(1+\sqrt{1-x})$<br>► or we prove that $g^{-1}(x) = f(x)$<br>►or we prove that $(g \circ f)(x) = x$ or $(f \circ g)(x) = x$</td><td>1</td></tr>
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| | 5 | $g'(x) = \dfrac{-1}{2\sqrt{1-x}\,(1+\sqrt{1-x})}$<br>The point A(ln2 , 0) of (C) is symmetric of B(0 , ln2 ) of (G).<br>The slope of the tangent at A to (C) is $f\,'(\ln 2) = \dfrac{1}{g\,'(0)} = -4$ | 1 |