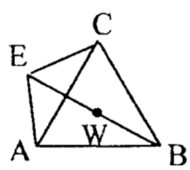
دورة سنة 2003 العادية	مسابقة في مادة الرياضيات اسس التصحيح	فرع علوم الحياة

Questions		ANSWERS	scheme
I -	1	$z' = \dfrac{\left(1+\frac{\sqrt{3}}{2}\right)+i\left(\frac{1}{2}+\sqrt{3}\right)-i}{2-i} = \frac{1}{2}+i\frac{\sqrt{3}}{2}$ $e^{i\frac{\pi}{3}} = \cos\frac{\pi}{3}+i\sin\frac{\pi}{3} = z'$	1
	2	EF = $\lvert z_F - z_E\rvert = \lvert 2-i\rvert = \sqrt{5}$ and FG = GE = $\sqrt{5}$ ► or $\lvert z'\rvert = 1 = \frac{EG}{EF}$ and arg z' = $(\overrightarrow{EF};\overrightarrow{EG}) = \frac{\pi}{3}$ hence EFG is equilateral.	1
	3	$\lvert z - z_E\rvert = \sqrt{5}$ then EM = $\sqrt{5}$, thus the set of points M is the circle (T) of center E and radius $\sqrt{5}$. EF = $\sqrt{5}$, F belongs to (T). ► or let z = x + iy, then $\lvert x+iy-i\rvert = \sqrt{5}$ which gives an equation of (T) : $x^2 + (y-1)^2 = 5$	1
II-	1	A(2 ; 0 ; 0) , B(0 ; 2 ; 0) and c(0 ; 0 ; 2)	½
	2	$\overrightarrow{N_P}$ (1 ; 1 ; 1) is a director vector of (d) and (d) passes through O. (d) : x = m , y = m , z = m .	½
	3a	W(m ; m ; m) with W is a point (P), hence m + m + m = 2 , m = $\frac{2}{3}$ Thus W $(\frac{2}{3};\frac{2}{3};\frac{2}{3})$.	½
	3b	WA = WB = WC = $\frac{2\sqrt{6}}{3}$, then W is the center of the circle circumscribed about the triangle ABC. ► or OA = OB = OC and W is the orthogonal projection of O on (ABC), So WA = WB = WC	1
	4a	We verify that W is the mid point of [BE] .	½
	4b	The area of ABCE is double of the area of triangle EAB ; S(EAB) = $\frac{1}{2}\lvert\overrightarrow{AB}\wedge\overrightarrow{AE}\rvert = \frac{4\sqrt{3}}{3}u^2$. Area ABCE = $\frac{8\sqrt{3}}{3}u^2$. ► Or area (ABCE) = $\frac{1}{2}$ BE × AC . 	1
III-	1a	P(X = 70 000) = (drawing 50 000 from L.P) × P(drawing 20 000 from R.P) P(X = 70 000) = $\frac{3}{5}\times\frac{3}{4} = \frac{9}{20}$	½

<table>
<tr><td rowspan="5">III-</td><td>1b</td><td>
<table>
<tr><td>x_i</td><td>40 000</td><td>70 000</td><td>120 000</td><td>150 000</td></tr>
<tr><td>P_i</td><td>$\frac{2}{5}\times\frac{3}{4}=\frac{3}{10}$</td><td>$\frac{9}{20}$</td><td>$\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$</td><td>$\frac{3}{5}\times\frac{3}{4}=\frac{3}{10}$</td></tr>
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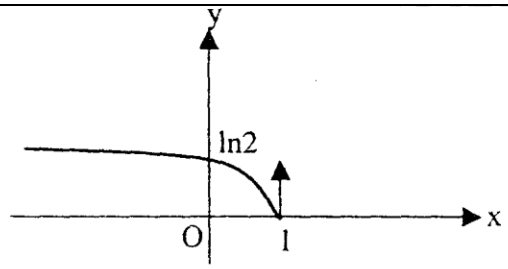
</td><td>1½</td></tr>
<tr><td>1c</td><td>$P(X < 100\,000) = \frac{6}{20}+\frac{9}{20}=\frac{15}{20}=\frac{3}{4}$.</td><td>½</td></tr>
<tr><td>2a</td><td>$P(S/D) = \frac{C_3^2}{C_4^2} = \frac{3}{6} = \frac{1}{2}$</td><td>2</td></tr>
<tr><td>2b</td><td>$P(S\cap D)=P(D)\times P(S/D)=\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$
$P(S/D) = \frac{C_2^2+C_2^1\times C_3^1}{C_5^2}=\frac{1+6}{10}=\frac{7}{10}$
$P(S\cap D)=\frac{1}{3}\times\frac{7}{10}=\frac{7}{30}$.
$P(S) = P(S\cap D) + P(S\cap G) = \frac{1}{3}+\frac{7}{30}=\frac{17}{30}$</td><td>2</td></tr>
<tr><td rowspan="6">IV-</td><td>1a</td><td>Using the representative curve, we can conform that the function f is continuous and strictly decreasing. Therefore it has an inverse function..</td><td>½</td></tr>
<tr><td>1b</td><td>$D_g =]-\infty\ ;1]$, (G) is symmetric of (C) for $x \geq 0$ with respect to the line of equation $y = x$.
</td><td>2</td></tr>
<tr><td>2</td><td>The characteristic equation associated to the differential equation is:
$r^2 - 3r + 2 = 0$, which has as solution $r_1 = 1$, $r_2 = 2$ we get : $y = C_1e^x + C_2e^{2x}$
we have $y(0) = 1$ and $y(\ln 2) = 0$ (or $y(0) = 1$ and $y'(0) = 0$), consequently
$C_1 = 2$ and $C_2 = -1$. Hence $f(x) = 2e^x - e^{2x}$</td><td>2</td></tr>
<tr><td>3</td><td>$A = \int_0^{\ln 2}(2e^x - e^{2x})\,dx = \left(2e^x - \frac{1}{2}e^{2x}\right)\Big|_0^{\ln 2} = 2e^{\ln 2} - \frac{1}{2}e^{2\ln 2} - 2 + \frac{1}{2} = 0{,}5u^2$</td><td>1½</td></tr>
<tr><td>4</td><td>$y = 2e^x - e^{2x}$, is equivalent to $2e^x - e^{2x} + y = 0$, $(e^x - 1)^2 = 1 - y$
$x \geq 0$ leads to $e^x \geq 1$ then $e^x - 1 = \sqrt{1-y}$, and $x = \ln(1+\sqrt{1-y})$
$g(x) = \ln(1+\sqrt{1-x})$
► or we prove that $g^{-1}(x) = f(x)$
►or we prove that $(g\circ f)(x) = x$ or $(f\circ g)(x) = x$</td><td>1</td></tr>
<tr><td>5</td><td>$g'(x) = \frac{-1}{2\sqrt{1-x}\,(1+\sqrt{1-x})}$
The point A(ln2 , 0) of (C) is symmetric of B(0 , ln2) of (G).
The slope of the tangent at A to (C) is $f'(\ln 2) = \frac{1}{g'(0)} = -4$</td><td>1</td></tr>
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