وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات شهادة الثانوية العامة فرع علوم الحياة	دورة سنة ٢٠٠٣ الاستثنائية
عدد المسائل: اربع	مسابقة في الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة) .

I – (3,5 points)

In the space referred to an orthonormal system $(O; \vec{i}\,; \vec{j}; \vec{k}\,)$, consider the points
A (2 ; l ; l) , B(0 ; 2 ; 3) , C (-1 ; 0 ; 5) et D (1 ; -l ; 3).

1) Write an equation of the plane (ABC) and verify that the point D belongs to this plane.
2) Give a system of parametric equations of the straight line (d) passing through B and D.
3) a- Prove that the quadrilateral ABCD is a rhombus.
   b- Calculate the distance from point A to the line (d).

II – (3.5 points)

In the complex plane, referred to a direct orthonormal system $(O; \vec{u}\,; \vec{v})$
consider the points A, B , M and M' of respective affixes 1, 5 , z and z'
where $z' = \dfrac{z-5}{z-1}$ $(z \neq 1)$

1) a- Give a geometric interpretation of $|z-5|$, $|z-1|$ and $|z'|$
   b- Determine the set of points M when M' moves on the circle with center O and radius 1.
2) Let z = x + iy and z' = x' + iy'
   a- Express x' and y' in terms of x and y.
   b- On which line does the point M move, so that z' becomes real?

III – (5 points)

A- The table below shows the distribution of the ages of the 40 employees in a certain factory:

Age in years	[20 ; 30[	[30 ; 40[	[40 ; 50[	[50 ; 60]
Frequency	13	12	10	5

1) Calculate the average age of these 40 employees.
2) a- Draw the increasing cumulative frequency polygon.
   b- The line of equation y = 20 cuts this polygon at a point M.
   Calculate the abscissa of M and give a significance of the obtained value.

B- The 40 employees of this factory are : 5 engineers, 10 technicians and 25 workers, distributed according to the following table:

	[20 ; 30[	[30 ; 40[	[40 ; 50[	[50 ; 60]
engineer	1	1	2	1
technician	2	3	3	2
worker	10	8	5	2

A group of 3 employees is chosen randomly from this factory.
Consider the following events:
E : «***the chosen group consists of an engineer, a technician and a worker***».
G : «***the chosen group is formed of 3 employees each having an age greater than or equal to 40 years***».

1) Calculate the probabilities: P(E) and P(G).
2) Calculate P(E/G) and $P(E \cap G)$.

IV – (8 points)

Let f be the function defined on $]0; +\infty[$, by $f(x) = \frac{1}{2}x + \frac{1+\ln x}{x}$ and (C) be its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1) Prove that the line of equation x = 0 is an asymptote of (C).
2) a- Calculate $\lim_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = \frac{1}{2}x$ is an asymptote of (C).
   b- Determine the coordinates of E, the point of intersection of the line (d) with the curve (C).
3) Verify that $f'(x) = \frac{x^2 - 2\ln x}{2x^2}$
4) The adjacent table is the table of variations of the function h defined by :
   $h = x^2 - 2\ln x$

x	0		1		$+\infty$
h'(x)	‖	−	0	+	
h(x)	‖ $+\infty$	↘	1	↗	$+\infty$

   a- Verify that f is strictly increasing on $]0; +\infty[$
   b- Consider, on the curve (C), a point W of abscissa 1
      Write an equation of the line (D) tangent to (C) at the point.
5) Draw the curve (C) and the lines (d) and (D) and plot the points E and W.
6) Calculate the area of the region bounded by the curve (C), the asymptote (d) and the straight lines of equations x = 1 and x = e.

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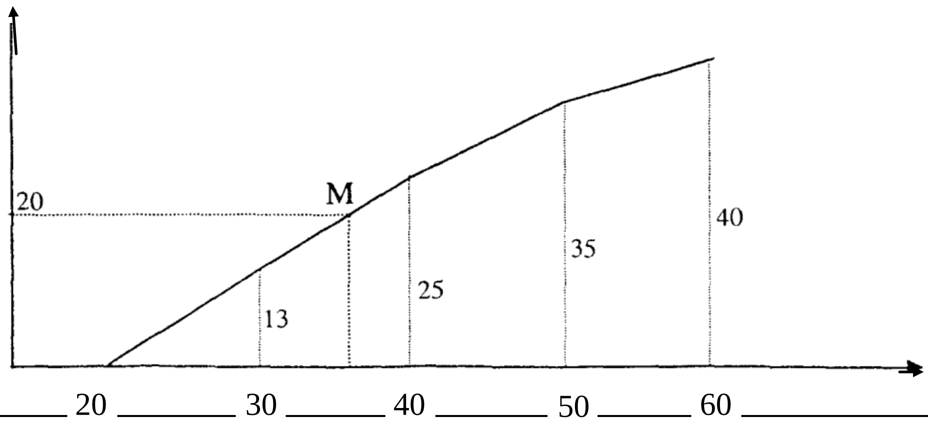
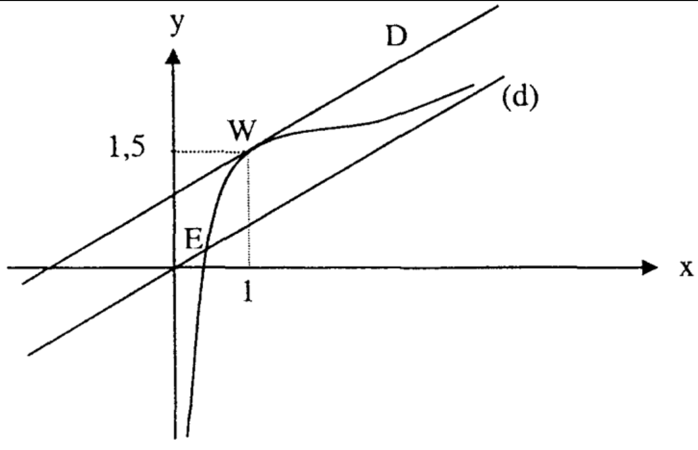
Questions		Elements de Réponses	Notes
I -	1	$\overrightarrow{AM}.\left(\overrightarrow{AB}\wedge\overrightarrow{AC}\right)=0$; $6x+2y+5z-19=0$. $6X_D+2y_D+5z_D-19=6(1)+2(-1)+5(3)-19=0$.	1
	2	$x=\lambda$; $y=-3\lambda+2$, $z=3$	½
	3(a)	$\overrightarrow{AB}\,(-2;1;2)$ et $\overrightarrow{DC}\,(-2;1;2)$ donc $\overrightarrow{AB}=\overrightarrow{DC}$, ABCD est un parallélogramme, de plus AB = BC = 3 , c'est donc un losange . ⊳ ou : AB = BC = CD = DA = 3 . ⊳ ou : [AC] et [BD] ont même milieu et $\overrightarrow{AC}.\overrightarrow{BD}=0$.	1
	3(b)	Les diagonales du losange sont perpendiculaires et ont le même milieu H : Distance (A ; (d)) = AH = $\frac{1}{2}$ AC = $\frac{\sqrt{26}}{2}$. ⊳ ou ; Distance (A ; (d)) = $\frac{\left\Vert\overrightarrow{AB}\wedge\overrightarrow{BD}\right\Vert}{\left\Vert\overrightarrow{BD}\right\Vert}$.	1
II-	1(a)	$\lvert z-5\rvert=\lvert z_M-z_B\rvert=BM$ $\lvert z-1\rvert=\lvert z_M-z_A\rvert=AM$ et $\lvert z'\rvert=OM'$.	1
	1(b)	OM' = 1 donc MB = MA , l'ensemble des points M est la médiatrice de [AB].	1
	2(a)	$x'+iy'=\frac{x-5+iy}{x-1+iy}=\frac{(x-5+iy)(x-1-iy)}{(x-1)^2+y^2}=\frac{x^2+y^2-6x+5+4iy}{(x-1)^2+y^2}$ $x'=\frac{x^2+y^2-6x+5}{(x-1)^2+y^2}$ et $y'=\frac{4y}{(x-1)^2+y^2}$	1
	2(b)	z' est un réel donc y' = 0 par suite y = 0 et M se déplace sur l'axe des abscisses.	½
III-	A(1)	L'âge moyen est égal à 36,75 ans.	½
	A(2-a)	(table and graph below)	1

Age	[20 ; 30[	[30 ; 40[	[40 ; 50[	[50 ; 60]
E.C.C.	13	25	35	40



	A(2-b)	$\frac{x_M - 30}{40-30} = \frac{20-13}{25-13}$ $x_M = 35{,}8$. 35,8 est la médiane de cette série ou 50 % des employés de cette usine ont un âge inférieur ou égal (ou supérieur ou égal) à 35,8 ans.	1
	B(1)	P(E) = $\frac{C_5^1\, C_{10}^1\, C_{25}^1}{C_{40}^3} = \frac{1250}{9880}$ = 0,126 et P(G) = $\frac{C_{15}^3}{C_{40}^3} = \frac{455}{9880}$ = 0,046	1
	B(2)	P(E/G) = $\frac{C_3^1\, C_5^1\, C_{25}^1}{C_{15}^3} = \frac{105}{455}$ = 0,23 et P(E$\cap$G) = P(G) × P(E/G) = 0,01	1 ½
	1	$\lim\limits_{x \to 0^+} f(x) = 0 + \frac{1-\infty}{0^+} = -\infty$.	½
	2(a)	$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(\frac{1}{2}x + \frac{1}{x} + \frac{\ln x}{x} \right) = +\infty.$ $\lim\limits_{x \to +\infty} \left(f(x) - \frac{1}{2}x \right) = \lim\limits_{x \to +\infty} \left(\frac{1}{x} + \frac{\ln x}{x} \right) = 0.$	1
	2(b)	f(x) = $\frac{1}{2}$ x donc 1 + lnx = 0 , d'où x = $\frac{1}{e}$ et E ($\frac{1}{e}$; $\frac{1}{2e}$).	1
	3	f '(x) = $\frac{1}{2} + \frac{1-1-\ln x}{x^2} = \frac{x^2 - 2\ln x}{2x^2}$.	½
	4(a)	f '(x) = $\frac{h(x)}{2x^2}$ avec h(x) $\geq$ 1 donc f '(x) > 0 et f est strictement croissante.	½
	4(b)	Y = (x – 1) f '(1) + f(1) = $\frac{1}{2}$ x + 1 .	1
	5		2
	6	A = $\int_1^e \frac{1+\ln x}{x} dx = \left[\frac{(1+\ln x)}{2} \right]_1^e = \frac{3}{2} u^2$.	1 ½

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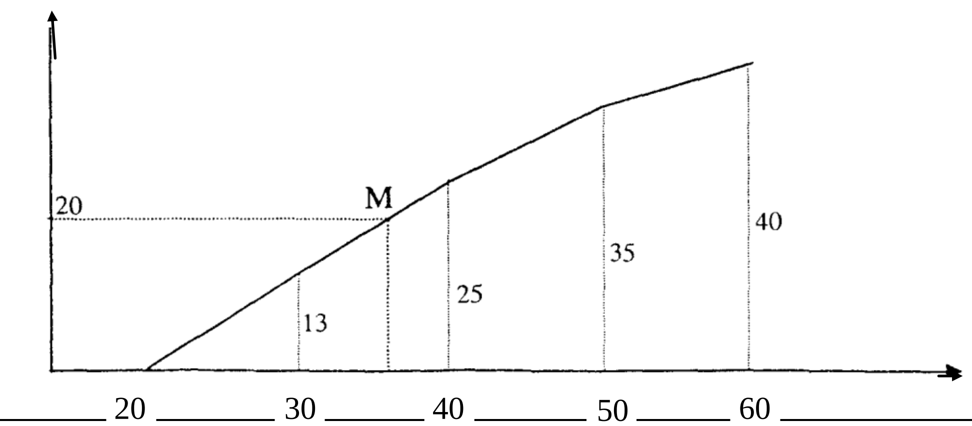
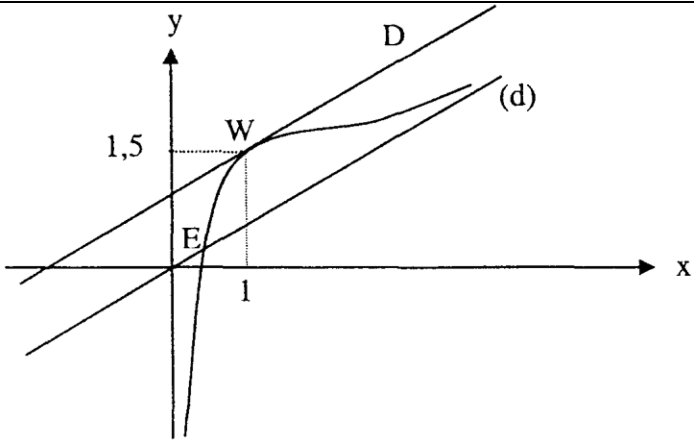
Questions		Answers	Marks
I -	1	$\overrightarrow{AM}.\left(\overrightarrow{AB} \wedge \overrightarrow{AC}\right) = 0$; $6x + 2y + 5z - 19 = 0$. $6X_D + 2y_D + 5z_D - 19 = 6(1) + 2(-1) + 5(3) - 19 = 0$.	1
	2	$x = \lambda$; $y = -3\lambda + 2$, $z = 3$	½
	3(a)	$\overrightarrow{AB}$ (–2; 1; 2) and $\overrightarrow{DC}$ (–2; 1; 2) then $\overrightarrow{AB} = \overrightarrow{DC}$, ABCD is a parallelogram; AB = BC = 3 , it is a rhombus. ▷ Or : AB = BC = CD = DA = 3 ▷ Or : [AC] et [BD] have the same mid point and $\overrightarrow{AC}.\overrightarrow{BD} = 0$.	1
	3(b)	The diagonals of the rhombus are perpendicular and they have the same mid point H : Distance (A ; (d)) = AH = $\frac{1}{2}$ AC = $\frac{\sqrt{26}}{2}$. ▷ Or ; Distance (A ; (d)) = $\frac{\left\Vert \overrightarrow{AB} \wedge \overrightarrow{BD} \right\Vert}{\left\Vert \overrightarrow{BD} \right\Vert}$.	1
II-	1(a)	$\lvert z - 5 \rvert = \lvert z_M - z_B \rvert = BM$ $\lvert z - 1 \rvert = \lvert z_M - z_A \rvert = AM$ and $\lvert z' \rvert = OM'$	1
	1(b)	OM' = 1 then MB = MA , the set of points M is the perpendicular bisector of [AB] .	1
	2(a)	$x' + iy' = \frac{x - 5 + iy}{x - 1 + iy} = \frac{(x - 5 + iy)(x - 1 - iy)}{(x-1)^2 + y^2} = \frac{x^2 + y^2 - 6x + 5 + 4iy}{(x-1)^2 + y^2}$ $x' = \frac{x^2 + y^2 - 6x + 5}{(x-1)^2 + y^2}$ et $y' = \frac{4y}{(x-1)^2 + y^2}$	1
	2(b)	z' is real so y' = 0 then y = 0 and M moves on the axes of abscissas.	½
III-	A(1)	The average age is equal to 36.75 years.	½
	A(2-a)	<table><tr><td>Age</td><td>[20 ; 30[</td><td>[30 ; 40[</td><td>[40 ; 50[</td><td>[50 ; 60]</td></tr><tr><td>I.C.F.</td><td>13</td><td>25</td><td>35</td><td>40</td></tr></table> 	1

	A(2-b)	$\frac{x_M - 30}{40-30} = \frac{20-13}{25-13}$ $x_M = 35.8$ 35,8 is the median of this distribution or 50 % of the employees of this factory have an age less than or equal to (greater than or equal to) 35,8 years.	1
	B(1)	P(E) = $\frac{C_5^1\, C_{10}^1\, C_{25}^1}{C_{40}^3} = \frac{1250}{9880}$ = 0.126 and P(G) = $\frac{C_{15}^3}{C_{40}^3} = \frac{455}{9880}$ = 0.046	1
	B(2)	P(E/G) = $\frac{C_3^1\, C_5^1\, C_{25}^1}{C_{15}^3} = \frac{105}{455}$ = 0.23 and P($E \cap G$) = P(G) × P(E/G) = 0.01	1 ½
	1	$\lim\limits_{x \to 0^+} f(x) = 0 + \frac{1-\infty}{0^+} = -\infty$.	½
	2(a)	$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(\frac{1}{2}x + \frac{1}{x} + \frac{\ln x}{x} \right) = +\infty$. $\lim\limits_{x \to +\infty} \left(f(x) - \frac{1}{2}x \right) = \lim\limits_{x \to +\infty} \left(\frac{1}{x} + \frac{\ln x}{x} \right) = 0$.	1
	2(b)	f(x) = $\frac{1}{2}$ x then 1 + lnx = 0 , So x = $\frac{1}{e}$ and E ($\frac{1}{e}$; $\frac{1}{2e}$) .	1
	3	f '(x) = $\frac{1}{2} + \frac{1-1-\ln x}{x^2} = \frac{x^2 - 2\ln x}{2x^2}$.	½
	4(a)	f '(x) = $\frac{h(x)}{2x^2}$ with h(x) $\geq$ 1 then f '(x) > 0 and f is strictly increasing.	½
	4(b)	Y = (x – 1) f '(1) + f(1) = $\frac{1}{2}$ x + 1 .	1
	5		2
	6	A = $\int_1^e \frac{1+\ln x}{x}\,dx = \left[\frac{(1+\ln x)}{2} \right]_1^e = \frac{3}{2}u^2$.	1 ½