| وزارة التربية والتعليم العالي | امتحانات شهادة الثانوية العامة | |
|---|---|---|
| المديرية العامة للتربية | | |
| دائرة الامتحانات | فرع علوم الحياة | دورة سنة 2004 العادية |

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| | المدة : ساعتان | الرقم : |

**ملاحظة :** يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

**I - (3 points)**

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A and B such that : $z_A = 1$ and $z_B = \frac{3}{2} + i\frac{\sqrt{3}}{2}$.

Let (C) be the circle with center A and radius 1.

1) a - Write $z_B - z_A$ in the exponential form.

b – Determine a measure of the angle $(\vec{u}\,;\overrightarrow{AB})$.

c – Show that the point B belongs to the circle (C).

2) To every point M, of non-zero affix Z, associate the point M' of affix z' such that $z' = \frac{\bar{z} + 2}{\bar{z}}$.

a – Prove that $\bar{z}\,(z' - 1) = 2$.

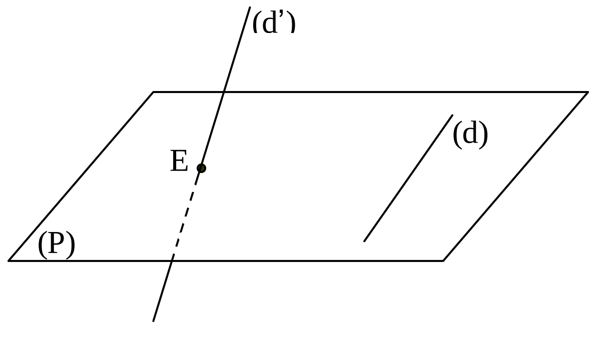
b – Deduce, when M' moves on the circle (C), that M moves on a circle (T) to be determined.

**II - (4 points)**

In the space referred to a direct orthonormal system $(O\,; \vec{i}, \vec{j}, \vec{k})$, consider the lines (d) and (d') defined by :

$$(d):\begin{cases} x = t+1 \\ y = 2t \\ z = t-1 \end{cases} \quad \text{and} \quad (d'):\begin{cases} x = 2m \\ y = -m+1 \\ z = m+1 \end{cases}$$

( t and m are two real parameters).




1) Prove that (d) and (d') are skew (not coplanar).

2) a – Show that x – y + z = 0 is an equation of the plane (P) determined by O and (d).

b – Determine the coordinates of E, the point of intersection of (P) and (d').

c – Prove that the straight line (OE) cuts (d).

3) a – Calculate the distance from point O to the line (d).

b – Deduce that the circle in plane (P), with center O and passing through E, is tangent to line (d).

**III- (9 points )**

Let f be the function defined, on $]0\,;+\infty[$ by $f(x)=x+2\dfrac{\ln x}{x}$ . (C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$ ; unit 2 cm .

1) a – Calculate $\lim\limits_{x\to 0} f(x)$ and give its graphical interpretation.

   b – Determine $\lim\limits_{x\to +\infty} f(x)$ and verify that the line (d) of equation y = x is an asymptote of (C).

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   c – Study according to the values of x , the relative position of ( C ) and (d) .

2) The table below is the table of variations of the function f ', the derivative of f.

| $x$ | $0$ | | $e$ | | $e\sqrt{e}$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $f''(x)$ | ‖ | $-$ | | $-$ | $0$ | $+$ | |
| $f'(x)$ | ‖ $+\infty$ | $\searrow$ | $1$ | $\searrow$ | $1-e^{-3}$ | $\nearrow$ | $1$ |

   a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.

   b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e .

   c – Prove that the curve(C) has a point of inflection L.

   d – Show that the equation f(x) = 0 has a unique root $\alpha$ and verify that $0{,}75< \alpha < 0{,}76$ .

3) Draw (D) , (d) and (C) .

4) Calculate, in $cm^2$ , the area of the region bounded by the curve (C), the line (d) and the two lines of equations x = 1 et x = e .

**IV- (4points)**

Consider two urns U and V :

U contains **three** balls numbered 0 and **two** balls numbered 1 .

V contains **five** balls numbered 1 to 5 .

**A -** One ball is drawn randomly from each urn.

Designate by X the random variable that is equal to the product of the two numbers that are marked on the two drawn balls.

1) Prove that P(X = 0) is equal to $\dfrac{3}{5}$ .

2) Determine the probability distribution of X .

**B** – In this part, the 10 balls that were in urns U and V are all placed in one urn W .

Two balls are drawn, simultaneously and at random, from this urn W.

1) What is the number of possible draws of these 2 balls?

2) Let q designate the product of the too numbers that are marked on the two drawn balls. .

   a - Show that the probability P(q = 0) is equal to $\dfrac{8}{15}$ .

   b – Calculate the probability P(q < 4) .