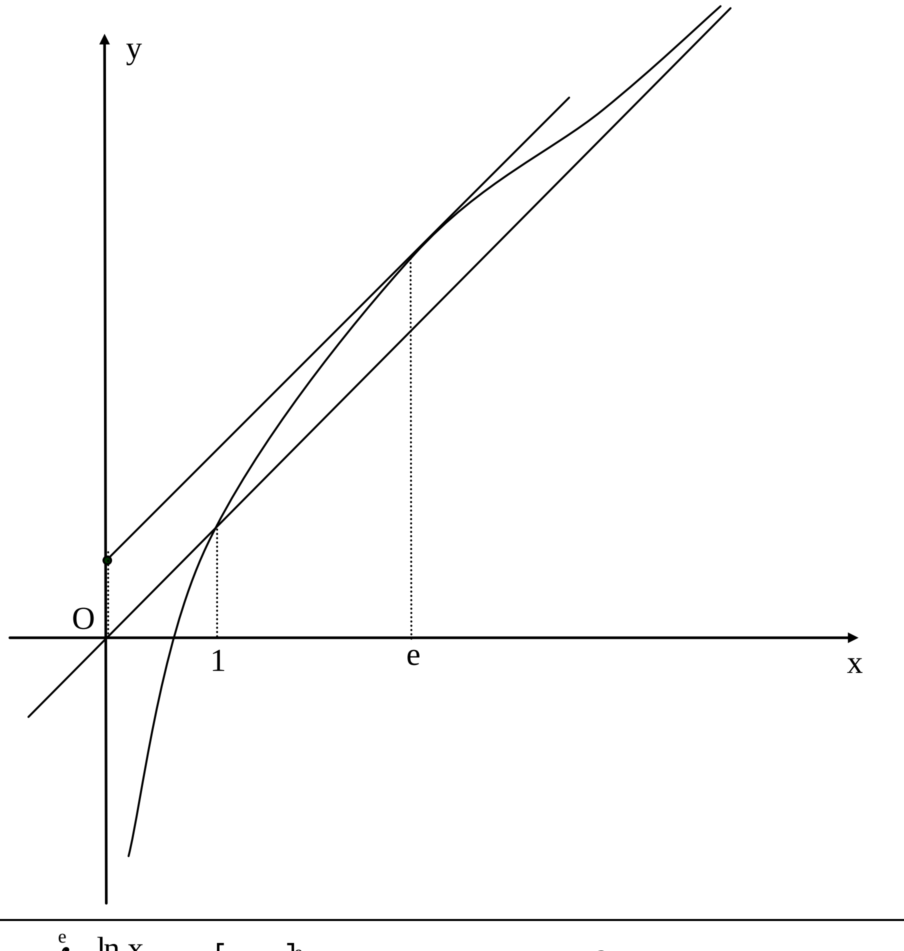
Life sciences		MATH	1st SESSION 2004
Q		Answers	M
I	1-a	$z_B - z_A = \frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$	
	1-b	$(\vec{u}\,;\vec{AB}) = \arg(Z_{\vec{AB}}) = \arg(z_B - z_A) = \frac{\pi}{3}$	
	1-c	$AB = \lvert z_B - z_A \rvert = 1$ then B belongs to (C).	
	2-a	$\bar{z}(z'-1) = \bar{z}(\frac{\bar{z}+2}{\bar{z}} - 1) = \bar{z}\left(\frac{2}{\bar{z}}\right) = 2$.	
	2-b	If M' moves on (C) then AM' = 1 and $\lvert z'-1 \rvert = 1$ hence $\lvert \bar{z} \rvert = 2$ then $\lvert z \rvert = 2$ and M moves on the circle of center O and radius 2.	

II	1	$\vec{V}(1\,;2\,;1)$ and $\vec{V'}(2\,;-1\,;1)$; $\vec{V}$ and $\vec{V'}$ are not collinear, then (d) and (d') are not parallel. Study of the intersection of (d) and (d') : t + 1 = 2m ; 2t = – m + 1 ; t – 1 = m + 1 Take 2t = – m + 1 ; t – 1 = m +1, we get t = 1 and m = –1, these values do not verify t + 1 = 2m. Hence (d) and (d') are skew ► Or : Let L (1 ; 0 ; –1) be a point of (d) and J (2 ; 0 ; 2) be a point of (d') ; $\vec{LJ}.(\vec{V} \wedge \vec{V'}) = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix} = -12 \neq 0$	
	2-a	By verification : O is a point of (P) (d) lies in (P) because t + 1 –2t + t – 1 = 0 for every real number t. ► Or : M(x ; y ; z) belongs to (P) iff $\vec{OM}.(\vec{OL} \wedge \vec{V}) = 0$ which gives x – y + z = 0	
	2-b	2m + m –1 + m + 1 = 0 ; m = 0 then E (0 ; 1 ; 1).	
	2-c	(OE) is a line in plane (P), (OE) and (D) are coplanar and they are not parallel ($\vec{OE}$ and $\vec{V}$ are not collinear), therefore they intersect. ► Or : Determine a system parametric equations of (OE) and then prove that it cuts (d).	
	3-a	distance (O/ (d)) = ….. = $\sqrt{2}$.	
	3-b	OE = $\sqrt{2}$ = distance (O/ (d)) ; then (C) is tangent to (d).	

III	1-a	$\lim\limits_{x\to 0} \ln x = -\infty$ then $\lim\limits_{x\to 0} f(x) = -\infty$; y'y is an asymptote of (C).	
	1-b	$\lim\limits_{x\to +\infty} \frac{\ln x}{x} = 0$ then $\lim\limits_{x\to +\infty} f(x) = +\infty$; $\lim\limits_{x\to +\infty} [f(x) - x] = 0$ hence the line (d) of equation $y = x$ is an asymptote of (C) at $+\infty$.	
	1-c	$f(x) - x = 2\frac{\ln x}{x}$. For x = 1, (C) cuts (d). For $0 < x < 1$, $f(x) - x < 0$ then (C) is below (d). For $x > 1$, (C) is above (d).	
	2-a	$f'(x) \ge 1 - \frac{1}{e^3} > 0$ then f is strictly increasing. $\begin{array}{c\|l} x & 0 \qquad\qquad\qquad\qquad +\infty \\ \hline f(x) & \| -\infty \nearrow +\infty \end{array}$	
	2-b	$y = f'(e)(x - e) + f(e)$; $y = x - e + e + \frac{2}{e} = x + \frac{2}{e}$	
	2-c	f ''(x) vanishes for $x = e\sqrt{e}$ and changes sign, then (C) has a point of inflection L of abscissa $e\sqrt{e}$.	
	2-d	f is continuous and changes sign on its domain, f(x) = 0 has at least a root α, moreover f is strictly increasing, then α is unique. $f(0.75) \times f(0.76) = -0.017 \times 0.377 < 0$, then $0.75 < \alpha < 0.76$.	
	3		
	4	$A = \int_1^e 2\frac{\ln x}{x}\,dx = \left[\ln^2 x\right]_1^e = 1\,u^2$, then A = 4 cm².	

IV	A-1	To get a product equal to 0 it's enough to draw from U a ball numbered 0, therefore the probability is equal to $\frac{3}{5}$. ►Or : Number of possible draws is equal to $5 \times 5 = 25$ $P(X = 0) = \frac{3\times 5}{5\times 5} = \frac{3}{5}$.	
	A-2	<table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>p_i</td><td>3/5</td><td>2/25</td><td>2/25</td><td>2/25</td><td>2/25</td><td>2/25</td></tr></table>	
	B-1	$C_{10}^{2} = 45$.	
	B-2 a	To get a product equal to 0 we must obtain one of the following outcomes: Two balls numbered 0 or {0 ; a} with a = 1, 2, 3, 4, 5. Number of favorable cases is $C_3^2 + C_3^1 \times C_7^1 = 24$ $P(q = 0) = \frac{24}{45} = \frac{8}{15}$	
	B-2 b	$P(q < 4) = P(q = 0) + P(q = 1) + P(q = 2) + P(q = 3)$ $= \frac{8}{15} + \frac{C_3^2 + C_3^1 \times C_1^1 + C_3^1 \times C_1^1}{45} = \frac{33}{45} = \frac{11}{15}$.	