

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة سنة 2004 الاستثنائية
عدد المسائل : اربع	مسابقة في الرياضيات المدة : ساعتان	الاسم : الرقم :

**ملاحظة:** يُسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.  
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

### I- (3.5 points).

In the plane referred to a direct orthonormal system  $(O; \vec{u}, \vec{v})$ , consider the points A , B and M of affixes -1 , 4 and z respectively, and let M' be the point of affix z' such that

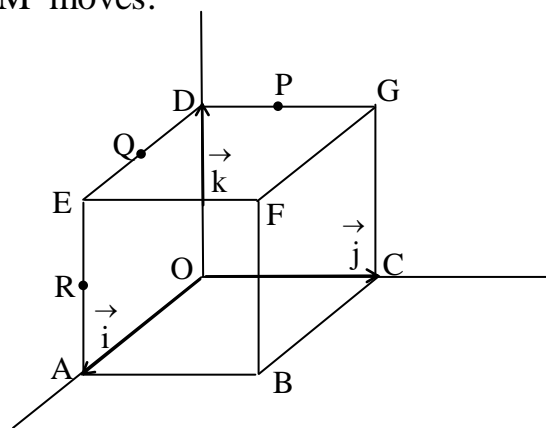
$$z' = \frac{z - 4}{z + 1} \quad (z \neq -1).$$

- 1) In the case where  $z = 1 + i$  , write  $z'$  in its algebraic form, and give its exponential form.
- 2) Determine the values of z for which  $z' = z$  .
- 3) a- Give a geometric interpretation of  $|z + 1|$  , and of  $|z - 4|$  .  
b- Find, when  $|z'| = 1$ , the line on which the point M moves.

### II- (3.5 points).

In the space referred to a direct orthonormal system

$(O; \vec{i}, \vec{j}, \vec{k})$ , consider the cube OABCDEFG such that : A(1 ; 0 ; 0) , B(1 ; 1 ; 0) and F(1 ; 1 ; 1) .  
Designate by P, Q and R the midpoints of the segments [DG] , [DE] and [AE] respectively .



- 1) a- Show that  $2x + 2y + 2z - 3 = 0$  is an equation of the plane (PQR).  
b- Prove that the plane (PQR) passes through the midpoint of [AB] .  
c- Prove that the planes (PQR) and (BEG) are parallel.
- 2) a- What is the nature of quadrilateral EGCA ?  
b- Let M be a variable point on the line (AC) .

$$\text{Show that } \vec{AM} \times \vec{EF} = \vec{AM} \times \vec{GF}.$$

### III- ( 4 points).

A multiple choice test is made up of **three** independent questions. The candidate is required to answer all the questions .Each question has two suggested answers out of which only one is correct.

**A candidate answers randomly each of these three questions.**

- 1) a- Show that the probability that he answers the three questions correctly is equal to  $\frac{1}{8}$  .

b- Consider the event  $E$  : « Among the three answers of the candidate, exactly two are correct » .

Calculate the probability of  $E$ .

2) The test is marked as follows : **+5** points for each correct answer, and **-3** points for each wrong answer.

Designate by  $X$  the random variable that is equal to the total mark obtained by the candidate upon answering the questions of this test.

a- Determine the 4 possible values of  $X$  .

b- Determine the probability distribution of  $X$  , and calculate the mean (expected value)  $E(X)$ .

#### IV-( 9 points).

Consider the differential equation (E) :  $y'' - 2y' + y = x + 1$ .

1) Let  $y = z + x + 3$  .

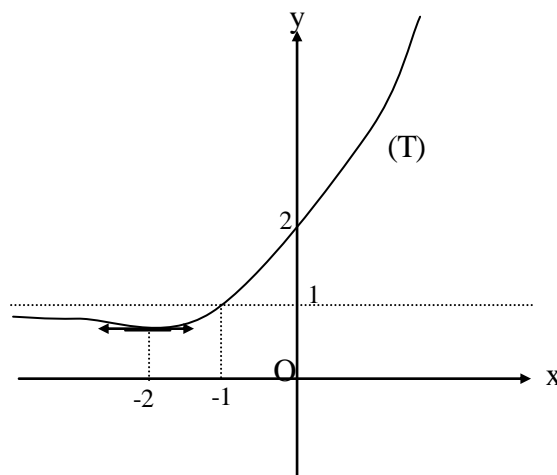
a- Write a differential equation (E') satisfied by  $z$ , and solve (E').

b- Deduce the general solution of (E).

2) Let  $f$  be a particular solution of (E) .

The curve (T), in the adjacent figure, is the representative curve of the function  $f'$  **the derivative** of  $f$  .

Show that  $f(x) = xe^x + x + 3$ .



**Designate by (C) the representative curve of the function  $f$  in an orthonormal**

**system  $(O; \vec{i}, \vec{j})$  ; unit 2cm .**

3) a- Calculate  $f(1)$  and  $\lim_{x \rightarrow +\infty} f(x)$  .

b- Calculate  $\lim_{x \rightarrow -\infty} f(x)$  , and show that the line (d) of equation  $y = x + 3$  is an asymptote of (C) .

c- Determine, according to the values of  $x$ , the relative positions of (C) and (d) .

d- Verify that  $I(-2; 1 - \frac{2}{e^2})$  is a point of inflection of the curve (C) .

4) a- Verify that  $f$  is strictly increasing on  $\mathbb{R}$  , and set up its table of variations.

b- Draw (d) and (C).

c- Calculate ,in  $\text{cm}^2$ , the area of the region bounded by the curve (C), the line (d) and the lines of equations  $x = 0$  and  $x = 1$  .