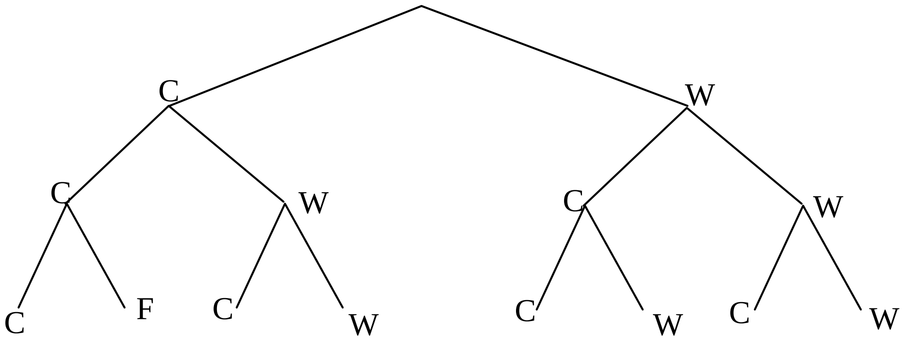
<table>
<tr><td colspan="2">Life Sciences</td><td>MATH</td><td>2^{nd} Session 2004</td></tr>
<tr><td colspan="2">Questions</td><td>Answers</td><td>G</td></tr>
<tr><td rowspan="4">I</td><td>1</td><td>$z' = \frac{1+i-4}{1+i+1} = \frac{-3+i}{2+i} = -1 + i = \sqrt{2}e^{i\frac{3\pi}{4}}$</td><td>1 ½</td></tr>
<tr><td>2</td><td>$z' = z$; $z = \frac{z-4}{z+1}$; $z^2 = -4$; $z = -2i$ or $z = 2i$.</td><td>½</td></tr>
<tr><td>3-a-</td><td>$|z + 1| = |z_M - z_A| = MA$; $|z - 4| = |z_M - z_B| = MB$</td><td>½</td></tr>
<tr><td>3-b-</td><td>$|z'| = \frac{MB}{MA}$; since $|z'| = 1$ then MB = MA.
M moves on the perpendicular bisector of [AB].</td><td>1</td></tr>
<tr><td rowspan="5">II</td><td>1-a-</td><td>P(0 ; ½ ; 1) , Q(½ ; 0 ; 1) , R(1 ; 0 ; ½)
the coordinates of P , Q and R verify the equation $2x + 2y + 2z - 3 = 0$
▶ or : M(x ; y ; z) is a point of the plane (PQR) iff $\overrightarrow{PM}.(\overrightarrow{PQ} \wedge \overrightarrow{PR}) = 0$.</td><td>½</td></tr>
<tr><td>1-b-</td><td>I(1 ; ½ ;0) : mid point of [AB], the coordinates of I satisfy the equation of the plane (PQR).</td><td>½</td></tr>
<tr><td>1-c-</td><td>(PQ) is parallel to (EG) ;
(QR) is parallel to (DA) which is parallel to (BG)
(PQR) contains two intersecting lines parallel to two intersecting lines in de (EBG) ; then (PQR) and (EBG) are parallel.
▶or: $x + y + z - 2 = 0$ is an equation of the plane (BEG) .
The two distinct planes(PQR) and (BEG) are parallel having two collinear normal vectors.</td><td>1</td></tr>
<tr><td>2-a-</td><td>$\overrightarrow{EA} = \overrightarrow{GC}$
(EA) is perpendicular to the plane (OABC) , then $(EA)\perp(AC)$
EAGC is a rectangle.</td><td>½</td></tr>
<tr><td>2-b-</td><td>$\overrightarrow{AM} \wedge \overrightarrow{EF} = \overrightarrow{AM} \wedge (\overrightarrow{EG} + \overrightarrow{GF}) = \overrightarrow{AM} \wedge \overrightarrow{EG} + \overrightarrow{AM} \wedge \overrightarrow{GF}$
$\overrightarrow{AM}$ and $\overrightarrow{EG}$ are collinear, $\overrightarrow{AM} \wedge \overrightarrow{EG} = \overrightarrow{0}$,then $\overrightarrow{AM} \wedge \overrightarrow{EF} = \overrightarrow{AM} \wedge \overrightarrow{GF}$
▶or
(AC) : $x = -\alpha + 1$; $y = \alpha$, $z = 0$
$\overrightarrow{AM}(-\alpha ; \alpha ; 0)$, $\overrightarrow{EF}(0 ;-1 ;0)$,$\overrightarrow{GF}(1 ;0 ;0)$; $\overrightarrow{AM} \wedge \overrightarrow{EF} = \overrightarrow{AM} \wedge \overrightarrow{GF} = \alpha \overrightarrow{k}$</td><td>1</td></tr>
<tr><td rowspan="5">III</td><td></td><td>
 </td><td></td></tr>
<tr><td>1-a-</td><td>P(CCC) = ½ × ½ ×½ = 1/8</td><td>1</td></tr>
<tr><td>1-b-</td><td>P(E) = P(CCW) + p(CWC) + p(WCC) = 1/8 +1/8 + 1/8 = 3/8</td><td>1</td></tr>
<tr><td>2-a-</td><td>The four possible values of X are – 9 ; – 1 ; 7 ; 15 .</td><td>½</td></tr>
<tr><td>2-b-</td><td><table><tr><td>$x = x_i$</td><td>–9</td><td>– 1</td><td>7</td><td>15</td></tr><tr><td>P_i</td><td>1/8</td><td>3/8</td><td>3/8</td><td>1/8</td></tr></table>
E(X) = – 9/8 – 3/8 + 21/8 + 15/8 = 3</td><td>1 ½</td></tr>
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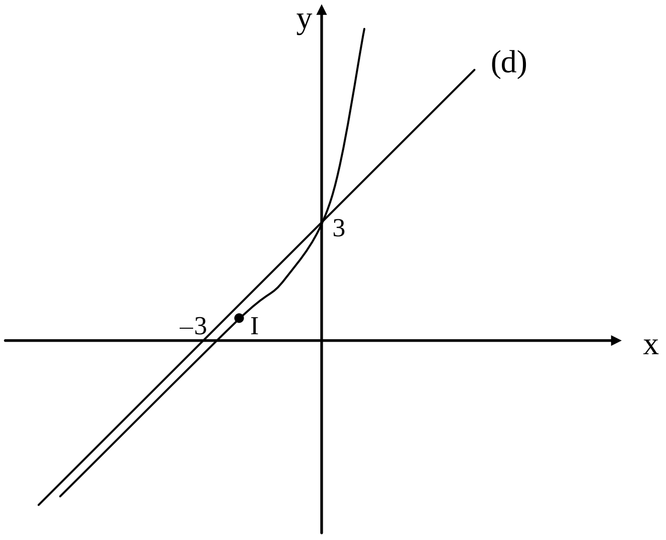
IV	1-a-	$y''-2y'+y=x+1$ with $y'=z'+1$ and $y''=z''$ then $z''-2z'+z=0$. Characteristic equation $r^2-2r+1=0$; $r_1=r_2=1$ and $z=(c_1x+c_2)e^x$.	1 ½
	1-b-	The general solution of (E) is $y=(c_1x+c_2)e^x+x+3$.	½
	2	According to the graph: $f'(-1)=1$ and $f'(0)=2$ $f'(x)=c_1e^x+(c_1x+c_2)e^x+1$ $f'(-1)=1$ gives $\frac{c_2}{e}+1=1$ so $c_2=0$, $f'(0)=2$ gives $c_1+c_2+1=2$ so $c_1=1$, $f(x)=xe^x+x+3$.	1
	3-a-	$f(1)=e+4\approx 6.738$, $\lim\limits_{x\to+\infty} f(x)=+\infty$.	½
	3-b-	$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty} xe^x+\lim\limits_{x\to-\infty}(x+3)=0-\infty=-\infty$. $\lim\limits_{x\to-\infty}[f(x)-(x+3)]=\lim\limits_{x\to-\infty} xe^x=0$, then the line (d) of equation y = x + 3 is an asymptote of (C) .	1
	3-c-	$f(x)-(x+3)=xe^x$ For x = 0 , (C) cuts (d) at point (0 ; 3) For x > 0 , (C) is above (d) For x < 0 , (C) is below (d) .	1
	3-d-	According to the graph $f''(-2)=0$, Over]– ∞ ;–2[: f ’ is decreasing then f ‘’ (x) < 0 Over]–2 ; +∞[: f ’ is increasing then f ‘’ (x) > 0 The point I(–2 ;f(–2)= $1-\frac{2}{e^2}$) is a point of inflection of (C).	½
	4-a-	(T) is above the axis of abscissas , then f ’(x) > 0 for every x , hence f is strictly increasing over R $\begin{array}{c\|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$	½
	4-b-		1 ½
	4-c-	A = $\int_0^1 xe^x dx=(x-1)e^x\Big\|_0^1=1\ u^2$ then A = 4 cm².	1