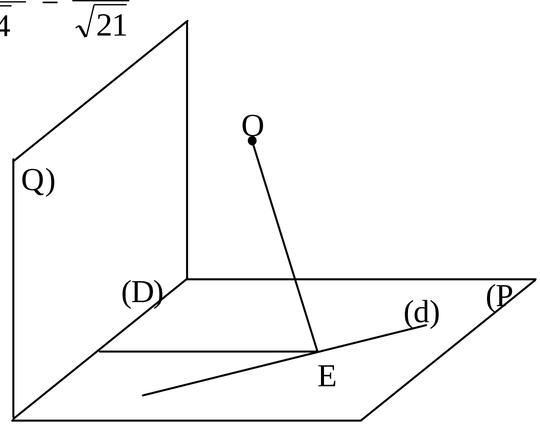
L.S.		*MATH*	1st session 2005
Question		Short Answers	M
I	1	$z' = i(\sqrt{2}\, e^{i\frac{\pi}{4}}) + 1 + i = i(1+i) + (1+i) = 2i$.	½
	2	$1+\sqrt{3} + 2i = iz + 1 + i$; $iz = \sqrt{3} + i$; $z = 1 - i\sqrt{3}$; $\lvert z\rvert = 2$ and $\arg(z) = -\frac{\pi}{3}$	½
	3	$z' = z$ for $z = iz + 1 + i$; $z(1-i) = 1+i$; $z = \frac{1+i}{1-i}$; $z = i$.	½
	4a	$z' - i = iz + 1 = i(z - i)$	½
	4b	$\lvert z' - i\rvert = \lvert i\rvert\,\lvert z - i\rvert = \lvert z - i\rvert$; EM' = EM. M moves on the circle (C) , EM = 3 , then EM' = 3, thus M' moves on the same circle.	1

II	1	Every point $M(2t+1; -t-1; t)$ on (d) is a point in (P) because $2(2t+1) - t - 1 - 3t - 1 = 0$; $0t=0$, hence (d) is included in (P).	½
	2	$I(1, -1, 0)$ is a point on (d), the equation of (S) is given by : $\vec{OM}.(\vec{OI} \wedge \vec{V_d}) = 0$; $\begin{vmatrix} x & y & z \\ 1 & -1 & 0 \\ 2 & -1 & 1 \end{vmatrix} = 0$; $x + y - z = 0$.	1
	3	E is a point on (d) (for $t = -½$). $\vec{OE}.\vec{V_d} = 0 + ½ - ½ = 0$. ►OR : Find the coordinates of the orthogonal projection of O on (d).	1
	4a	$\vec{n}_P(2; 1; -3)$ and $\vec{n}_Q(1; 4; 2)$; $\vec{n}_P . \vec{n}_Q = 0$; (P) and (Q) are perpendicular.	½
	4b	(P) and (Q) are perpendicular, E is a point in (P) ; $d(E/(D)) = d(E/(Q)) = \frac{\lvert 0-2-1+1\rvert}{\sqrt{1+16+4}} = \frac{2}{\sqrt{21}}$ ►OR : Find a system of Parametric equations of (D) and calculate the distance from E to (D). 	1

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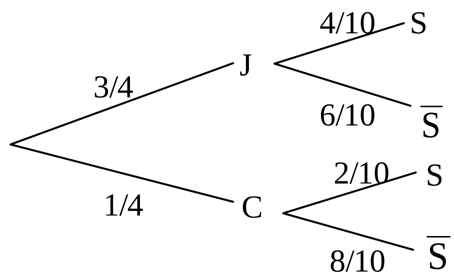
III			
	1a	$P(S\cap J) = P(J) \times P(S/J) = \frac{3}{4}\times\frac{4}{10} = \frac{3}{10}$.	½
	1b	$P(S\cap C) = P(C) \times P(S/C) = \frac{1}{4}\times\frac{2}{10} = \frac{1}{20}$. $P(S) = P(S\cap J) + P(S\cap C) = \frac{6}{20} + \frac{1}{20} = \frac{7}{20}$. $P(C/S) = \frac{P(C\cap S)}{P(S)} = \frac{1/20}{7/20} = \frac{1}{7}$. $P(C/\overline{S}) = \frac{P(C\cap\overline{S})}{P(\overline{S})} = \frac{P(C)\times P(\overline{S}/C)}{1-P(S)} = \frac{(1/4)(8/10)}{1-(7/20)} = \frac{4}{13}$.	1½
	2a	A customer has bought only one out of the following four choices : only a jacket, only a coat, a jacket and a shirt, a coat and a shirt. $X(\Omega)$ = { 150 000 , 200 000 , 210 000 , 260 000}	½
	2b	$\begin{array}{c\|c\|c\|c\|c} X = x_i & 150\,000 & 200\,000 & 210\,000 & 260\,000 \\ \hline P_i & \frac{3}{4}\times\frac{6}{10}=\frac{18}{40} & \frac{1}{4}\times\frac{8}{10}=\frac{8}{40} & \frac{3}{4}\times\frac{4}{10}=\frac{12}{40} & \frac{1}{4}\times\frac{2}{10}=\frac{2}{40} \end{array}$	1 ½
	2c	$E(X) = \frac{10000}{40}(15 \times 18 + 20 \times 8 + 21 \times 12 + 26 \times 2) = 183\,500$	½
	2d	The sales amount during that week is equal to the product of the mean amount by the number of the customers : 183 500 × 120 = 22 020 000 LL.	½

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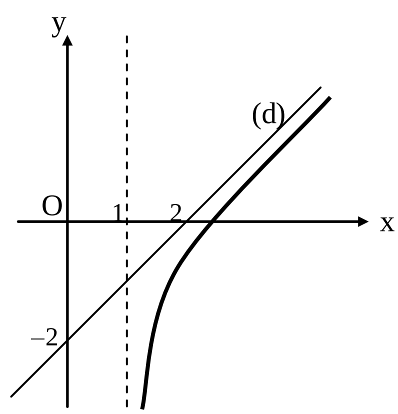
IV	1a	$\lim\limits_{\substack{x\to1\\x>1}} e^x = e$; $\lim\limits_{\substack{x\to1\\x>1}} (e^x - e) = 0^+$; $\lim\limits_{\substack{x\to1\\x>1}} f(x) = -\infty$ The line of equation x = 1 is an asymptote to (C) .	½
	1b	$\lim\limits_{x\to+\infty} \dfrac{3e^x}{e^x - e} = 3$, consequently $\lim\limits_{x\to+\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty} [f(x) - (x-2)] = \lim\limits_{x\to+\infty} [3 - \dfrac{3e^x}{e^x - e}] = 0$ The line (d) of equation $y = x - 2$ is an asymptote to(C).	1
	1c	$f(x) - (x-2) = 3 - \dfrac{3e^x}{e^x - e} = \dfrac{-3e}{e^x - e}$ $x > 1$, $e^x > e$, then $f(x) - (x-2) < 0$ so (C) is below (d).	½
	2	$f'(x) = 1 - 3\dfrac{e^x(e^x - e) - e^x(e^x)}{(e^x - e)^2} = 1 + 3\dfrac{e^{x+1}}{(e^x - e)^2} > 0$ x: 1 ‖ $+\infty$ f '(x): ‖ + f (x): ‖ $-\infty$ ↗ $+\infty$	1
	3	On I, f is continuos and changes signs, thus the equation f(x) = 0 has at least one root α. But since f is strictly increasing on I ,then α is unique. $f(2.6) = -0.158$ and $f(2.7) = 0.0294$, thus $2.6 < \alpha < 2.7$	1
	4		1
	5	• $\displaystyle\int_3^4 \frac{e^x}{e^x - e}dx = [\ln(e^x - e)]_3^4 = \ln(e^4 - e) - \ln(e^3 - e) = \ln\frac{e^3 - 1}{e^2 - 1}$. • $\mathcal{A} = \displaystyle\int_3^4 (x - 2 - f(x))dx = \int_3^4 (-3 + 3\frac{e^x}{e^x - e})dx = [-3x]_3^4 + 3\ln\frac{e^3 - 1}{e^2 - 1}$ $= [-3 + 3\ln\dfrac{e^3 - 1}{e^2 - 1}]\ u^2 \approx 0.28\ u^2$	1 ½
	6a	On I, f being continuous and strictly increasing, it has an inverse function g.	½
	6b	The equation f(x) = g(x) is equivalent to f(x) = x, so $1 - \dfrac{3e^x}{e^x - e} = 0$ gives $2e^x = -e$ which is impossible. ► OR : graphically , the curve (C) does not cut the first bisector y = x.	1