

عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:
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**ملاحظة :** يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.  
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

### I- (4 points)

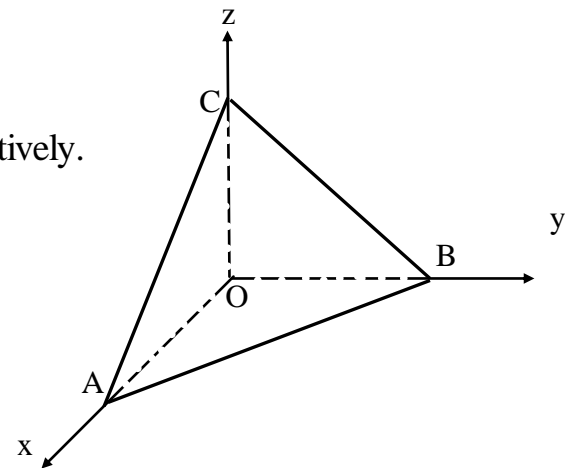
In the complex plane referred to a direct orthonormal system  $(O ; \vec{u}, \vec{v})$ , consider the points E and F of affixes  $z_E = \frac{\sqrt{3}+1}{4} - \frac{\sqrt{3}-1}{4}i$  and  $z_F = \frac{1}{2} + \frac{1}{2}i$ .

- 1) a- Calculate  $(z_E)^2$  and find the modulus and an argument of  $(z_E)^2$ .
  - b- Determine the modulus of  $z_E$  and verify that  $-\frac{\pi}{12}$  is an argument of  $z_E$ .
  - c- Deduce the exact values of  $\cos \frac{\pi}{12}$  and  $\sin \frac{\pi}{12}$ .
- 2) Let  $Z = \frac{z_E}{z_F}$ .
- a- Write  $z_E$ ,  $z_F$  and  $Z$  in the exponential form.
  - b- Show that the triangle OEF is equilateral.

### II- (4 points)

In the space referred to a direct orthonormal system  $(O ; \vec{i}, \vec{j}, \vec{k})$ , consider the points A (4 ; 0 ; 0), B (0 ; 4 ; 0) and C (0 ; 0 ; 4).

- 1) Write an equation of plane (ABC).
- 2) Calculate the area of triangle ABC.
- 3) Let F and G be the midpoints of [AC] and [BC] respectively.
  - a- Give a system of parametric equations of the straight line (FG).
  - b- The plane of equation  $z = 0$  intersects the plane (OFG) along a line (d). Prove that the lines (d) and (FG) are parallel to each other.
  - c- Calculate the distance between the two lines (d) and (AB).



### III- ( 4 points)

The 80 students of the third secondary classes in a certain school are distributed into the three sections GS , LS and SE as shown in the following table :

	GS	LS	SE
Girls	8	18	10
Boys	12	14	18

The school director chooses randomly a group of 3 students, from the third secondary classes, to participate in a TV program.

- 1) What is the number of possible groups?
- 2) Designate by  $X$  the random variable that is equal to the number of boys in the chosen group. Determine the probability distribution of  $X$  .
- 3) Show that the probability that the chosen group contains one girl from each section is  $\frac{18}{1027}$ .
- 4) The chosen group is made up of 3 girls, What is the probability that they are from the same section ?

### IV- (8 points)

Let  $f$  be the function that is defined on  $\mathbb{R}$  by :  $f(x) = x + 2 - e^{-x}$ , and  $(C)$  be its representative curve in an orthonormal system  $(O ; \vec{i}, \vec{j})$ .

- 1) a- Calculate  $\lim_{x \rightarrow +\infty} f(x)$  and prove that the line  $(d)$  of equation  $y = x + 2$  is an asymptote of  $(C)$ .  
b- Calculate  $\lim_{x \rightarrow -\infty} f(x)$  and give, in the decimal form, the values of  $f(-1.5)$  and  $f(-2)$ .
- 2) Calculate  $f'(x)$  and set up the table of variations of  $f$ .
- 3) Write an equation of the line  $(T)$  that is tangent to  $(C)$  at the point  $A$  of abscissa 0.
- 4) Show that the equation  $f(x) = 0$  has a unique root  $\alpha$  and verify that  $-0.5 < \alpha < -0.4$  .
- 5) Draw  $(d)$ ,  $(T)$  and  $(C)$  .
- 6) Designate by  $g$  the inverse function of  $f$ , on  $\mathbb{R}$ .
  - a- Draw, in the system  $(O ; \vec{i}, \vec{j})$ , the curve  $(G)$  that represents  $g$ .
  - b- Designate by  $A(\alpha)$  the area of the region that is bounded by the curve  $(C)$ , the axis of abscissas and the two lines of equations  $x = \alpha$  and  $x = 0$  .  
Show that  $A(\alpha) = (-\frac{\alpha^2}{2} - 3\alpha - 1)$  units of area.
  - c- Deduce the area of the region that is bounded by the curve  $(G)$ , the axis of abscissas and the two lines of equations  $x = 0$  and  $x = 1$  .