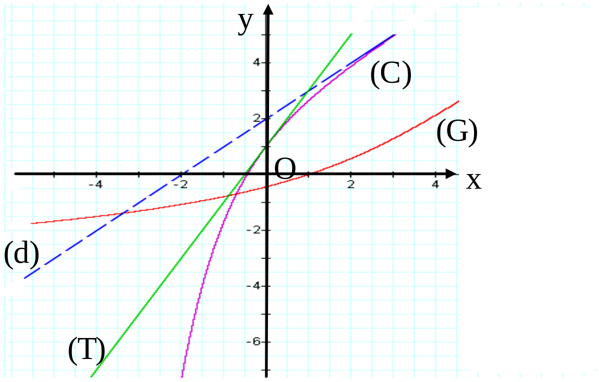
L.S-MATHS

2^{nd} session 2005

Q1	Short Answers	M
1.a	$z_E^2 = \frac{1}{16}(3+1+2\sqrt{3}-3-1+2\sqrt{3}-4i) = \frac{1}{16}(4\sqrt{3}-4i) = \frac{\sqrt{3}}{4}-\frac{1}{4}i$ $z_E^2 = \frac{1}{2}(\frac{\sqrt{3}}{2}-\frac{1}{2}i) = \frac{1}{2}e^{-i\frac{\pi}{6}}$; $\lvert z_E{}^2\rvert = \frac{1}{2}$; $\arg(z_E{}^2) = -\frac{\pi}{6}$.	1
1.b	$\lvert z_E{}^2\rvert = \frac{1}{2}$ then $\lvert z_E\rvert = \frac{1}{\sqrt{2}}$. $\arg(z_E{}^2) = 2\arg(z_E) = -\frac{\pi}{6}+2k\pi$; $\arg(z_E) = -\frac{\pi}{12}+k\pi$, since $R_e(z_E) > 0$ and $Im(z_E)<0$, therefore $\arg(z_E) = -\frac{\pi}{12}$.	1
1.c	$z_E = \frac{1}{\sqrt{2}}[\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12})] = \frac{1}{\sqrt{2}}[\cos(\frac{\pi}{12})-i\sin(\frac{\pi}{12})] = \frac{\sqrt{3}+1}{4}-\frac{\sqrt{3}-1}{4}i$ $\cos(\frac{\pi}{12}) = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $\sin(\frac{\pi}{12}) = \frac{\sqrt{6}-\sqrt{2}}{4}$.	½
2.a	$z_E = \frac{1}{\sqrt{2}}e^{-i\frac{\pi}{12}}$, $z_F = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}}$, $Z = e^{i(-\frac{\pi}{12}-\frac{\pi}{4})} = e^{-i\frac{\pi}{3}}$	½
2.b	$\lvert Z\rvert = 1 = \frac{OE}{OF}$; OE = OF $\arg(Z) = \arg(z_E) - \arg(z_F) = (\vec{u},\vec{OE}) - (\vec{u},\vec{OF})\ [2\pi] = (\vec{OF},\vec{OE})\ [2\pi] = -\frac{\pi}{3}[2\pi]$. OEF is equilateral. • OR : $EF = \lvert z_F - z_E\rvert = \frac{1}{\sqrt{2}} = OE = OF$	1

Q2	Short Answers	M
1	$\vec{AM}.(\vec{AB}\wedge\vec{AC}) = 0$; $\begin{vmatrix} x-4 & y & z \\ -4 & 4 & 0 \\ -4 & 0 & 4 \end{vmatrix} = 0$; $x + y + z - 4 = 0$	1
2	Area(ABC) = $\frac{1}{2}\lVert \vec{AB}\wedge\vec{AC}\rVert = 2\sqrt{3}\ u^2$	½
3.a	F(2 ;0 ;2) and G(0 ; 2 ; 0) , eq. of (FG) : $x = -2t$, $y = 2t + 2$, $z = 2$.	½
3.b	The plane of eq. z = 0 is the plane (AOB) ; (FG) // (AB) then (FG) // (OAB), since (d) is the line of intersection of (OFG) and (OAB) , hence (FG) //(d). • OR :the equation of (OFG) is : $x + y - z = 0$. (d) : $x = m$, $y = -m$, $z = 0$. $\vec{V_d}(1;-1;0)$, $\vec{FG}(-2;2;0)$ then $\vec{FG} = -2\vec{V_d}$ and the lines (d) and (FG) are distinct ; Therefore they are parallel.	1
3.c	The distance between (d) and (AB) is the distance from O to (AB) since (d) passes through O , and (d) //(AB), then $d = \frac{\lVert \vec{OA}\wedge\vec{OB}\rVert}{\lVert \vec{AB}\rVert} = 2\sqrt{2}$ u.	1

Q3	Short Answers	M
1	Number of possible cases : $C_{80}^{3} = 82\,160$.	½
2	$\begin{array}{c c c c c} \hline x_i & 0 & 1 & 2 & 3 \\ \hline p_i & \frac{C_{36}^{3}}{C_{80}^{3}} = \frac{7140}{82160} & \frac{C_{36}^{2} \times C_{44}^{1}}{C_{80}^{3}} = \frac{27720}{82160} & \frac{C_{36}^{1} \times C_{44}^{2}}{C_{80}^{3}} = \frac{34056}{82160} & \frac{C_{44}^{3}}{C_{80}^{3}} = \frac{13244}{82160} \\ \hline \end{array}$	1½
3	$\frac{C_{8}^{1} \times C_{18}^{1} \times C_{10}^{1}}{C_{80}^{3}} = \frac{18}{1027}$.	1
4	p(the girls are from the same section / 3 girls) $= \frac{C_{8}^{3} + C_{18}^{3} + C_{10}^{3}}{C_{36}^{3}} = \frac{248}{1785} = 0.138$	1

Q4	Short Answers	M
1.a	$\lim\limits_{x \to +\infty} f(x) = +\infty - 0 = +\infty$; $\lim\limits_{x \to +\infty} [f(x) - (x+2)] = \lim\limits_{x \to +\infty} (-e^{-x}) = 0$ then the line (d) of equation $y = x + 2$ is an asymptote of (C) .	1
1.b	$\lim\limits_{x \to -\infty} f(x) = -\infty - \infty = -\infty$; $f(-1.5) = -3.981$; $f(-2) = -7.389$.	1
2	$f'(x) = 1 + e^{-x}$ $\begin{array}{c c c c} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$	1
3	(T) : $y = f'(0)x + f(0)$; $y = 2x + 1$.	½
4	f is continuous, strictly increasing on IR and varies from $-\infty$ to $+\infty$, then the equation $f(x) = 0$ has a unique solution α. $f(-0.5) \times f(-0.4) = -0.148 \times 0.1081 < 0$ then $-0.5 < \alpha < -0.4$.	1
5		1½
6.a	See the figure.	½
6.b	$A(\alpha) = \int_{\alpha}^{0} f(x)\,dx = \int_{\alpha}^{0} (x + 2 - e^{-x})\,dx = \left[\frac{x^2}{2} + 2x + e^{-x}\right]_{\alpha}^{0} = 1 - \frac{\alpha^2}{2} - 2\alpha - e^{-\alpha}$ But $f(\alpha) = 0$ i.e. $\alpha + 2 - e^{-\alpha} = 0$, therefore $e^{-\alpha} = \alpha + 2$ and $A(\alpha) = (-1 - 3\alpha - \frac{\alpha^2}{2})\,.\,u^2$	1
6.c	The region bounded by the curve (G) , the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$, is symmetric of the preceding region with respect of the line of equation $y = x$, therefore the required area is equal to $A(\alpha)$.	½