

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:
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ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the plane (P) of equation $x + y + z - 4 = 0$ and the points A (3 ; 1 ; 0), B(1; 2 ; 1), C(1 ; 1 ; 2) and E(2 ; 0 ; -1).

- 1) Prove that the triangle ABC is right angled at B.
- 2) a- Verify that the points A, B and C belong to the plane (P).
b- Write a system of parametric equations of the line (d) that is perpendicular to plane (P) at point A, and verify that the point E belongs to (d).
- 3) Designate by (Q) the plane that passes through A and is perpendicular to (BE).
Write an equation of (Q).
- 4) The planes (P) and (Q) intersect along a line (D).
a- Prove that the lines (D) and (BC) are parallel.
b- M is a variable point on (BC), prove that the distance from M to plane (Q) remains constant.

II– (4.5 points)

A bag **S** contains **eight** bills: **four** bills of 10 000LL, **three** of 20 000LL and **one** of 50 000LL.
Another bag **T** contains also **eight** bills : **three** bills of 10 000LL and **five** of 20 000LL.

- 1) **Two bills are drawn, simultaneously and randomly, from the bag S.**

Calculate the probability of each of the following events:

A : « the two drawn bills are of the same category »

B : « the sum of values of the two drawn bills is 30 000LL ».

- 2) **One of the two bags S and T is randomly chosen, after which two bills are simultaneously and randomly drawn from this bag.**

Consider the following events:

E : « the chosen bag is **S** »

F : « the sum of values of the two drawn bills is 30 000LL »

Calculate the probabilities $P(F \cap E)$ and $P(F \cap \bar{E})$. Deduce $P(F)$.

- 3) **We draw, randomly, one bill from the bag S and one bill from the bag T.**

Let X be the random variable that is equal to the sum of the values of the two drawn bills.

a- Verify that $P(X = 60\,000) = \frac{3}{64}$.

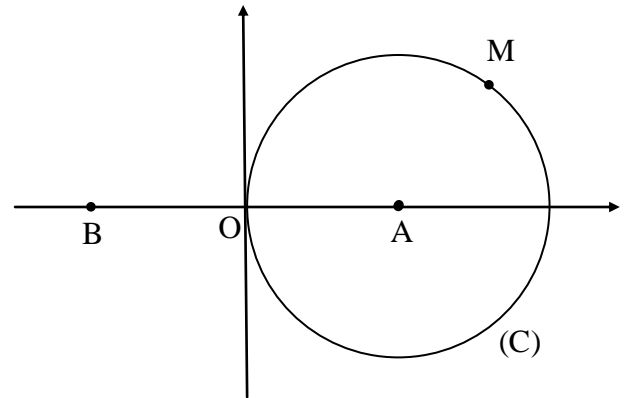
b- Determine the probability distribution of X and calculate its mean (expected value).

III– (3.5 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A and B of affixes 1 and -1 respectively. Let (C) be the circle of center A and of radius 1. The exponential form of the affix z of a point M on (C), other than O, is given by $z = re^{i\theta}$.

Let M' be the point of affix z' such that $z' = \frac{1}{r}e^{i(\pi+\theta)}$.

- 1) Show that $z' \times \bar{z} = -1$.
- 2) Show that the points O, M and M' are collinear.
- 3) a- Justify that $|z - 1| = 1$.
b- Prove that $|z' + 1| = |z'|$, and deduce that M' moves on a line (d) to be determined.
- 4) Determine the points M on (C) for which $z' = -z$.



IV– (8 points)

A- Consider the differential equation (E): $y'' - 4y' + 4y = 4x^2 - 16x + 10$.

Let $z = y - x^2 + 2x$.

- 1) Write a differential equation (E') satisfied by z .
- 2) Solve (E') and deduce the general solution of (E).
- 3) Determine the particular solution of (E) whose representative curve, in an orthonormal system, has at the point $A(0; 1)$ a tangent parallel to the axis of abscissas.

B- Let f be the function that is defined on \mathbb{R} by $f(x) = e^{2x} + x^2 - 2x$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

- 1) a- Calculate $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.
b- Calculate $f(1)$ and $f(-1.5)$ in their decimal forms.
- 2) The table below is the table of variations of the function f' , the derivative of f .

x	$-\infty$	0	$+\infty$
$f'(x)$	$-\infty$	0	$+\infty$

- a- Determine, according to the values of x , the sign of $f'(x)$.
- b- Set up the table of variations of f .
- 3) Draw the curve (C).
- 4) Let F be the function that is defined on $[0; +\infty[$ by $F(x) = \int_0^x f(t)dt$.
 - a- Determine the sense of variations of F .
 - b- What is the sign of $F(x)$? Justify your answer.