MATHEMATICS LS FIRST SESSION 2006

I	Answers	Marks
1	$\overrightarrow{BA}(2\,;-1\,;-1)$, $\overrightarrow{BC}(0\,;-1\,;1)$; $\overrightarrow{BA}\,.\,\overrightarrow{BC}=0$ then ABC is right at B	½
2.a	$x_A+y_A+z_A-4=3+1+0-4=0$; $A\in(P)$. $x_B+y_B+z_B-4=1+2+1-4=0$; $B\in(P)$. $x_C+y_C+z_C-4=1+1+2-4=0$; $C\in(P)$.	½
2.b	$\overrightarrow{N}_P(1;1;1)$ is a director vector of (d) then (d) : $x=\lambda+3$; $y=\lambda+1$; $z=\lambda$. For y = 0 we have $\lambda=-1$, then x = 2 and z = – 1, hence $E\in(d)$.	1
3	$\overrightarrow{BE}(1\,;-2\,;-2)$ is normal to (Q) , so (Q) : x – 2y – 2z + r = 0 . $A\in(Q)$ then r = – 1 ; (Q) : x – 2y – 2z – 1 = 0.	½
4.a	$(D)\begin{cases}x+y+z-4=0\\x-2y-2z-1=0\end{cases}$ $(D)\begin{cases}x=3\\y=-t+1\\z=t\end{cases}$ then $\overrightarrow{V}_D(0\,;-1\,;1)=\overrightarrow{BC}$; (BC) // (D). ▶**OR :**(BC) is perpendicular to (AB) and it is orthogonal to (EA), so (BC) is perpendicular to plane (EAB) and especially to (EB) , on the other hand since (EB) is perpendicular to (Q) then (BC) is parallel to (Q) . The plane (P), which contains (BC), cuts (Q) along (D) parallel to (BC).	1
4.b	$(BC):\begin{cases}x=1\\y=-m+2\\z=m+1\end{cases}$; M(1 ; – m + 2 ; m +1) ; $d(M;(Q))=\dfrac{\lvert 1+2m-4-2m-2-1\rvert}{\sqrt{1+4+4}}=2$. ▶**OR :** (BC) // (D) and $(D)\subset(Q)$,so (BC) // (Q) . Since $M\in(BC)$; $d(M;(Q))=cst$.	½

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<tr><th>II</th><th>Answers</th><th>Marks</th></tr>
<tr><td>1</td><td>P(A) = P[(10 000 , 10 000) or (20 000 , 20 000)] $=\dfrac{C_4^2+C_3^2}{C_8^2}=\dfrac{9}{28}$.
P(B) = P(10 000 , 20 000) = $\dfrac{C_4^1\times C_3^1}{C_8^2}=\dfrac{3}{7}$.</td><td>1</td></tr>
<tr><td>2</td><td>$P(F\cap E)=P(E)\times P(F/E)=\dfrac{1}{2}\times\dfrac{3}{7}=\dfrac{3}{14}$.
$P(F\cap\overline{E})=P(\overline{E})\times P(F/\overline{E})=\dfrac{1}{2}\times\dfrac{C_3^1\times C_5^1}{C_8^2}=\dfrac{1}{2}\times\dfrac{15}{28}=\dfrac{15}{56}$.
P(F) = $P(F\cap E)+P(F\cap\overline{E})=\dfrac{3}{14}+\dfrac{15}{56}=\dfrac{27}{56}$.</td><td>1½</td></tr>
<tr><td>3.a</td><td>P(X = 60 000) = P(50 000 , 10 000) = 1/8 × 3/8 = 3/64.</td><td>½</td></tr>
<tr><td>3.b</td><td>
<table>
<tr><td>$X=x_i$</td><td>20 000</td><td>30 000</td><td>40 000</td><td>60 000</td><td>70 000</td></tr>
<tr><td>p_i</td><td>$\dfrac{4}{8}\times\dfrac{3}{8}=\dfrac{12}{64}$</td><td>$\dfrac{4}{8}\times\dfrac{5}{8}+\dfrac{3}{8}\times\dfrac{3}{8}=\dfrac{29}{64}$</td><td>$\dfrac{3}{8}\times\dfrac{5}{8}=\dfrac{15}{64}$</td><td>$\dfrac{3}{64}$</td><td>$\dfrac{1}{8}\times\dfrac{5}{8}=\dfrac{5}{64}$</td></tr>
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E(X) = $(24+87+60+18+35)\times\dfrac{10\,000}{64}=35\,000$.</td><td>1½</td></tr>
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<tr><th>III</th><th>Answers</th><th>Marks</th></tr>
<tr><td>1</td><td>$z'\times\bar{z} = \frac{1}{r}e^{i(\pi+\theta)}\times re^{-i\theta} = e^{i\pi} = -1.$</td><td>½</td></tr>
<tr><td>2</td><td>$(\vec{u}\ ,\vec{OM}) = \theta$, $(\vec{u}\ ,\vec{OM'}) = \pi+\theta$, so $(\vec{OM}\ ,\vec{OM'}) = (\pi+\theta)-\theta = \pi$, hence O , M and M’ are collinear.
▶OR : $\frac{z'}{z} = -\frac{1}{r^2}$ **;** $z' = -\frac{1}{r^2}z$, so $\vec{OM'} = -\frac{1}{r^2}\vec{OM}$ and thus O , M , M’ are collinear</td><td>½</td></tr>
<tr><td>3.a</td><td>$|z-1| = |z_M - z_A| = AM = 1$</td><td>½</td></tr>
<tr><td>3.b</td><td>$|z' + 1| = \left|\frac{-1}{\bar{z}}+1\right| = \left|\frac{\bar{z}-1}{\bar{z}}\right| = \frac{|\overline{z-1}|}{|\bar{z}|} = \frac{|z-1|}{|\bar{z}|} = \frac{1}{|\bar{z}|}$ and $|z'| = \frac{|-1|}{|\bar{z}|} = \frac{1}{|\bar{z}|}$; $|z' +1| = |z'|$.
$|z_{M'} - z_B| = |z_{M'}|$; BM’ = OM’ ; M’ moves on the perpendicular bisector (d) of [OB].</td><td>1</td></tr>
<tr><td>4</td><td>$z' = -z$; $-z\times\bar{z} = -1$; $z\times\bar{z} = 1$; $|z|^2 = 1$; $OM^2 = 1$; OM = 1 , then M belongs to the circle (C’) of center O , radius 1; but M belongs to (C). Then points M are the two points of intersection of (C) and (C’).
▶OR: $|-z+1| = |-z|$; $|z-1| = |z|$; AM = OM ; M moves on the perpendicular bisector (D) of [OA].Then points M are the two points of intersection of (D) and (C).</td><td>1</td></tr>
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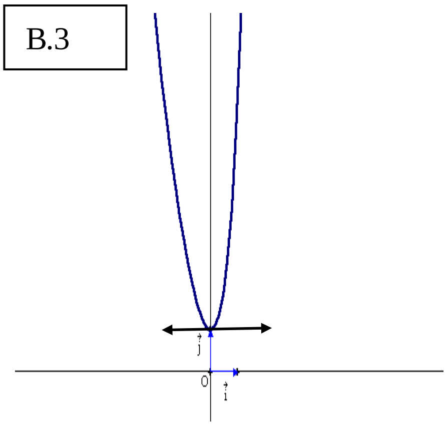
<tr><th>IV</th><th colspan="2">Answers</th><th>Marks</th></tr>
<tr><td>A.1</td><td colspan="2">$z' = y' - 2x + 2$ et $z'' = y'' - 2$
$z'' + 2 - 4(z' + 2x - 2) + 4(z + x^2 - 2x) = 4x^2 - 16x + 10$
$z'' - 4z' + 4z = 0$</td><td>½</td></tr>
<tr><td>A.2</td><td colspan="2">$r^2 - 4r + 4 = 0$; r = 2 double root ; $z = (Ax + B)e^{2x}$ and $y = (Ax + B)e^{2x} + x^2 - 2x$</td><td>1</td></tr>
<tr><td>A.3</td><td colspan="2">y(0) = 1 ; B = 1
y’(0) = 0 with $y'(x) = Ae^{2x} + 2(Ax + B)e^{2x} + 2x - 2$; A + 2B = 2 ; A = 0 ; $y = e^{2x} + x^2 - 2x$</td><td>1</td></tr>
<tr><td>B.1.a</td><td colspan="2">$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}[e^{2x} + x(x-2)] = +\infty$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}[e^{2x} + x(x-2)] = +\infty$</td><td>½</td></tr>
<tr><td>B.1.b</td><td colspan="2">f(1) = 6.39 ; f(–1.5) = 5.30 .</td><td>½</td></tr>
<tr><td>B.2.a</td><td colspan="2">f ‘(x) < 0 for x < 0 ; f ‘(x) > 0 for x > 0.</td><td>1</td></tr>
<tr><td>B.2.b</td><td>$$\begin{array}{c|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & +\infty & \searrow & 1 & \nearrow & +\infty \end{array}$$</td><td>B.3
 </td><td>B.2.b
½
B.3
1</td></tr>
<tr><td>B.4.a</td><td colspan="2">F’(x) = f(x) > 0 for x ≥ 0 (min (f(x)) =1) , then F is strictly increasing over [0 ; + ∞ [.</td><td>1</td></tr>
<tr><td>B.4.b</td><td colspan="2">f(t) > 0 and x ≥ 0 then $\int_0^x f(t)dt \geq 0$, so F(x) ≥ 0.
▶OR : F is increasing and F(0) = 0 , then F(x) ≥ 0.</td><td>1</td></tr>
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