

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:
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ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the points $A(1 ; 1 ; 0)$, $B(2 ; 0 ; 0)$, $C(1 ; 3 ; -1)$, $E(2 ; 2 ; 2)$ and the plane (P) of equation $x + y + 2z - 2 = 0$.

- 1) a- Verify that (P) is the plane determined by A, B and C.
b- Show that the line (AE) is perpendicular to the plane (P).
c- Calculate the area of triangle ABC and the volume of tetrahedron EABC.
- 2) Designate by L the midpoint of [AB] and by (Q) the plane passing through L and parallel to the two lines (AE) and (BC).
a- Write an equation of plane (Q).
b- Prove that the planes (P) and (Q) are perpendicular.
c- Prove that line (d), the intersection of the planes (P) and (Q), is parallel to (BC).

II- (4 points)

The 20 employees in a factory are distributed into two departments as shown in the table below:

	Technical Department	Administrative Department
Women	3	5
Men	10	2

- 1) The manager of this factory wants to offer a gift to one of the employees. To do this, he chooses randomly an employee of this factory.
Consider the following events:
W : « the chosen employee is a woman ».
M : « the chosen employee is a man ».
T : « the chosen employee is from the technical department ».
A : « the chosen employee is from the administrative department ».
a- Calculate the following probabilities:
 $P(W/T)$, $P(W/A)$, $P(W \cap T)$ and $P(W)$.
b- Knowing that the chosen employee is a man, what is the probability that he is from the technical department ?
- 2) On a different occasion, the factory manager chooses **two** employees randomly and simultaneously from the technical department and also chooses **one** employee randomly from the administrative department.
Designate by X the random variable that is equal to the number of women chosen.
a- Verify that $P(X=1) = \frac{95}{182}$.
b- Determine the probability distribution of X.

III– (4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points E, F, G of respective affixes $z_E = 2i$, $z_F = -2i$, $z_G = -1+i$ and let M be a point of affix z.

- 1) a- Find the set (T) of points M such that $|z-2i| = \sqrt{2}$.
b- Show that the point G belongs to (T).
- 2) a- Find the line (L) on which point M moves when $\left| \frac{z-2i}{z+2i} \right| = 1$.
b- Determine the affix z_0 of a point W on (L) such that $|z_0-2i| = 3$.
- 3) Let A and B be the points of respective affixes z_A and z_B such that:
 $z_A = z_F + z_G$ and $z_B = z_F \times z_G$.
a- Write the complex numbers z_A and z_B in the exponential form.
b- Prove that the points O, A and B are collinear.

IV– (8 points)

Consider the function f defined over $]-\infty, 0[\cup]0, +\infty[$ by $f(x) = x-1 - \frac{4}{e^x-1}$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

- 1) a- Show that the axis of ordinates is an asymptote to (C).
b- Calculate $\lim_{x \rightarrow +\infty} f(x)$ and prove that the line (d) of equation $y = x - 1$ is an asymptote to the curve (C).
c- Prove that the line (D) of equation $y = x + 3$ is an asymptote to (C) at $-\infty$.
- 2) Prove that the point S(0 ; 1) is a center of symmetry of (C).
- 3) a- Calculate $f'(x)$ and set up the table of variations of f.
b- Show that the equation $f(x) = 0$ has two roots α and β and verify that :
 $1.7 < \alpha < 1.8$ and $-3.2 < \beta < -3.1$.
- 4) Draw (d), (D) and (C).
- 5) a- Prove that $f(x) = x+3 - \frac{4e^x}{e^x-1}$.
b- Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations $x = 2$ and $x = 3$.
- 6) Let g be the inverse function of f on $]0, +\infty[$.
Prove that the equation $f(x) = g(x)$ has no roots.