وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع: علوم الحياة	دورة سنة 2008 الاكمالية الاستثنائية
عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the following table, only one of the proposed answers to each question is correct.
Write the number of each question and give, with justification, the corresponding answer.

N°	Questions	Answers a	b	c	d
1	If $\frac{\pi}{6}$ is an argument of z, then an argument of $\frac{i}{\bar{z}^2}$ is :	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$-\frac{5\pi}{6}$	$\frac{5\pi}{6}$
2	If $z=-\sqrt{3}+e^{i\frac{\pi}{6}}$, then the exponential form of z is:	$e^{\frac{5\pi}{6}i}$	$e^{\frac{7\pi}{6}i}$	$\sqrt{3}e^{-\frac{\pi}{6}i}$	$e^{-\frac{5\pi}{6}i}$
3	If z and z' are two complex numbers such that $\lvert z\rvert=2$ and $z'=z-\frac{1}{\bar{z}}$, then $\lvert z'\rvert=$	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
4	If z is a complex number with $\lvert z\rvert=\sqrt{2}$, then $\lvert \bar{z}+i\bar{z}\rvert=$	$2\sqrt{2}$	2	$\sqrt{2}$	$\frac{\sqrt{2}}{2}$

II- (4 points)

In the space referred to an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points:
A(0 ; 1 ; – 2), B(2 ; 1; 0), C(3 ; 0; – 3) and H(2 ; 2 ; – 2).

1) Show that $x-2y-z=0$ is an equation of the plane (P) determined by the points H , A and B and verify that the point C does not belong to this plane.

2) a- Show that triangle HAB is isosceles of vertex H.
   b- Show that (CH) is perpendicular to (P).
   c- Prove that CA = CB and determine a system of parametric equations of the interior bisector (δ) of angle ACB.

3) Let T be the orthogonal projection of H on plane (ABC).
   Prove that T belongs to (δ).

III- (4 points)

In order to prevent a certain disease, we vaccinated 40% of persons of a population.
Then we noticed that 85% of the vaccinated persons were not affected by the disease and that 75% of the persons who were not vaccinated are affected by the disease.
A person is chosen randomly from this population.
Consider the following events:

D : « the chosen person is affected by the disease».
V : « the chosen person is vaccinated ».

1) a- Verify that the probability of the event $D \cap V$ is equal to $\frac{6}{100}$.
b- What is the probability that the chosen person is affected by the disease and is not vaccinated?
c- Deduce the probability $P(D)$.

2) The chosen person is not affected by the disease.
Calculate the probability that he/she is vaccinated.

3) In this part, suppose that this population is formed of 300 persons.
We choose randomly 3 persons from this population.
What is the probability that at least one, among the 3 chosen persons, is vaccinated?

IV- (8 points)

Let f be the function defined over $]1;+\infty[$ by $f(x) = x - \frac{1}{x\ln x}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim_{x\to 1} f(x)$ and deduce an asymptote to (C).
2) Calculate $\lim_{x\to +\infty} f(x)$. Prove that the straight line (d) of equation $y = x$ is an asymptote to (C) and study the position of (C) and (d).

3) Calculate $f'(x)$ and show that f is strictly increasing.
Set up the table of variations of f.

4) Show that the equation $f(x) = 0$ has a unique root α and verify that $1.5 < \alpha < 1.6$.

5) Draw (d) and (C).

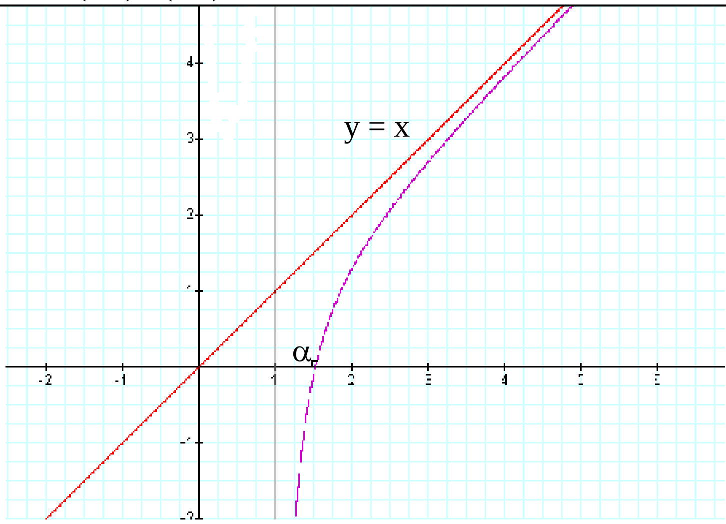
6) a- Calculate the area $A(t)$ of the region limited by the curve (C), the straight line (d) and the two straight lines of equations $x = e$ and $x = t$ where $t > e$.
b- Show that for all $t > e$, we have $A(t) < t$.

QI	Answer		Mark
1	$\arg\left(\frac{i}{\bar{z}^2}\right)=\arg(i)-2\arg(\bar{z})\ [2\pi]=\frac{\pi}{2}+2\left(\frac{\pi}{6}\right)\ [2\pi]=\frac{5\pi}{6}\ [2\pi]$	**d**	1
2	$z=-\sqrt{3}+\frac{\sqrt{3}}{2}+\frac{1}{2}i=-\frac{\sqrt{3}}{2}+\frac{1}{2}i=e^{i\left(\frac{5\pi}{6}\right)}$	**a**	1
3	$z'=\frac{z\bar{z}-1}{\bar{z}}=\frac{\lvert z\rvert^2-1}{\bar{z}}=\frac{3}{\bar{z}}$, so $\lvert z'\rvert=\frac{3}{\lvert z\rvert}=\frac{3}{2}$	**c**	1
4	$\lvert\bar{z}+i\bar{z}\rvert=\lvert\bar{z}(1+i)\rvert=\lvert\bar{z}\rvert\times\lvert 1+i\rvert=\sqrt{2}\times\sqrt{2}=2$	**b**	1

QII	Answer	Mark
1	$x_A - 2y_A - z_A = 0 - 2 + 2 = 0$; $x_B - 2y_B - z_B = 2 - 2 - 0 = 0$; $x_H - 2y_H - z_H = 2 - 4 + 2 = 0$, then $x - 2y - z = 0$ is an equation of the plane (P) determined by A , B and H. $x_C - 2y_C - z_C = 3 - 0 + 3 \neq 0$,then C does not belong to (P).	1
2a	$\overrightarrow{HA}(-2;-1;0)$; $\overrightarrow{HB}(0;-1;2)$ then $HA = HB = \sqrt{5}$.	0.5
2b	$\overrightarrow{HC}(1;-2;-1) = \overrightarrow{N}_{(P)}$ then (CH) is perpendicular to (P).	0.5
2c	Triangles AHC and BHC are congruent so CA = CB and triangle ABC is isosceles of vertex C (or CA = CB = $\sqrt{11}$) hence, the bisector of angle $A\hat{C}B$ is the median relative to the side [AB]. I(1 ; 1 ; – 1) is the midpoint of [AB]; $\overrightarrow{CI}(-2;1;2)$ is a direction vector of (δ) and $C\in(\delta)$. Thus, a system of parametric equations of (δ) is : $x = -2m + 3$; $y = m$ and $z = 2m - 3$.	1
3	(CH) is perpendicular to plane (P) then (CH) is orthogonal to the straight line (AB) in (P) ; the straight line (AB) being orthogonal to (CI) and (CH) , then (AB) is perpendicular to plane (CHI) , consequently planes (ABC) and (CHI) are perpendicular , Therefore T the foot of the perpendicular through H to plane (ABC) belongs to the straight line (CI) = (δ) , intersection of the two planes. •OR: $\overrightarrow{AB}\times\overrightarrow{AC}=2\vec{i}+8\vec{j}-2\vec{k}$ Then, plane (ABC) has an equation: $2x+8y-2z-12=0$ $(HT):\begin{cases}x=2t+2\\y=8t+2\\z=-2t-2\end{cases}$ $(HT)\cap(ABC)=\{T\}$ then $T\left(\frac{5}{3},\frac{2}{3},-\frac{5}{3}\right)$ T belongs to (δ) for $m=\frac{2}{3}$.	1

QIII	Answer	Mark
1a	$P(M\cap V) = P(V)\times P(M/V) = \frac{40}{100}\times\frac{15}{100}=\frac{6}{100}$.	0.5
1b	$P(M\cap\bar{V}) = P(\bar{V})\times P(M/\bar{V}) = \frac{60}{100}\times\frac{75}{100}=\frac{45}{100}$.	0.5

1c	$P(M) = P(M \cap V) + P(M \cap \overline{V}) = \frac{6}{100} + \frac{45}{100} = \frac{51}{100}$.	1
2	$P(V/\overline{M}) = \frac{P(V \cap \overline{M})}{P(\overline{M})} = \frac{\frac{40}{100} \times \frac{85}{100}}{1 - \frac{51}{100}} = \frac{34}{49}$.	1
3	Let A be the event : « at least one is vaccinated among the three persons » $P(A) = 1 - P(\overline{A}) = 1 - \frac{C_{180}^{3}}{C_{300}^{3}} = 0.785$.	1

QIV	Answer	Mark
1	$\lim\limits_{x \to 1^+} f(x) = 1 - \infty = -\infty$: the straight line of equation $x = 1$ is an asymptote to (C).	0.5
2	$\lim\limits_{x \to +\infty} f(x) = +\infty - 0 = +\infty$; $\lim\limits_{x \to +\infty} [f(x) - x] = 0$, then the straight line (d) of equation $y = x$ is an asymptote to (C). $f(x) - x = -\frac{1}{x \ln x} < 0$, so (C) is below (d).	1
3	$f'(x) = 1 + \frac{\ln x + 1}{x^2 \ln^2 x} > 0$ for $x > 1$, then f is strictly increasing . $$\begin{array}{c\|ccc} x & 1 & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$$	1.5
4	f is continuous and strictly increasing and f(x) increases from $-\infty$ to $+\infty$ then the equation $f(x) = 0$ has a unique root α. and $f(1.5) \times f(1.6) = -0.14 \times 0.27 < 0$ then $1.5 < \alpha < 1.6$.	1
5		1.5
6a	$A(t) = \int_e^t [x - f(x)].dx = \int_e^t \frac{1}{x \ln x}.dx = \int_e^t \frac{(\ln x)'}{\ln x}.dx = [\ln(\ln x)]_e^t = \ln(\ln t) - \ln(\ln e) = \ln(\ln t)$.	1.5
6b	$A(t) < t$ if $\ln(\ln t) < t$; $\ln t < e^t$ which is true since the representative curve of the ln function is below that of the exponential function.	1