دورة سنة 2009 العادية	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل : أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points M and M' of respective affixes z and z', where $z' = \left(1+i\sqrt{3}\right)z$.

1) In this part, suppose that z = 2i.

a- Determine the exponential form of z'.

b- Calculate $\left|\frac{z'}{z}\right|$ and $\arg\left(\frac{z'}{z}\right)$.

c- Show that triangle OMM' is right angled at M.

2) Assume in this part that $z = (1+i)^3$.

a- Write the exponential form and the algebraic form of z.

b- Write the exponential form and the algebraic form of z'.

c- Deduce the exact value of $\cos\frac{13\pi}{12}$.

II- (4 points)

Consider two bags $\mathbf{B_1}$ and $\mathbf{B_2}$ such that:

B_1 contains **six** cards numbered 1, 2, 3, 4, 5, 6.

B_2 contains **five** cards numbered 0, 1, 2, 4, 5.

A-

One card is drawn randomly from bag $\mathbf{B_1}$:

- if it carries one of the numbers 1 or 2, then **three** cards are drawn randomly and simultaneously from bag $\mathbf{B_2}$.
- But if it carries one of the numbers 3, 4, 5 or 6, then **two** cards are drawn randomly and simultaneously from bag $\mathbf{B_2}$.

Consider the following events:

K: « the card drawn from bag $\mathbf{B_1}$ carries the one of the numbers 1 or 2 ».

L: « the card drawn from bag $\mathbf{B_1}$ carries the one of the numbers 3, 4, 5 or 6 ».

E: « The product of numbers shown on the cards drawn from bag $\mathbf{B_2}$ is zero ».

1) a- Calculate the probabilities p(K) and p(L).

b- Show that $p(E\cap K) = \frac{1}{5}$.

c- Calculate $p(E\cap L)$ and deduce p(E).

2) Knowing that the product of the numbers shown on the cards drawn from bag $\mathbf{B_2}$ is zero, calculate the probability that **three c**ards were drawn from $\mathbf{B_2}$.

B-

In this part we use only the bag $\mathbf{B_2}$ and **three** cards are drawn randomly and simultaneously from this bag.
Let X be the random variable that is equal to the biggest number among those shown on the **three** drawn cards, thus the possible values of X 2, 4 and 5.

Prove that p(X= 4) = $\frac{3}{10}$, and determine the probability distribution of X.

III- (4 points)

In the space referred to a direct orthonormal system $\left(O\,;\vec{i}\,,\vec{j}\,,\vec{k}\right)$, consider the point A(1;0;1) and the two planes (P) and (Q) with equations $2x - y - 2 = 0$ and $x + 2y - z = 0$ respectively.

1) a- Verify that point A is a common to (P) and (Q).
   b- Determine a system of parametric equations of (d), the line intersection of (P) and (Q).

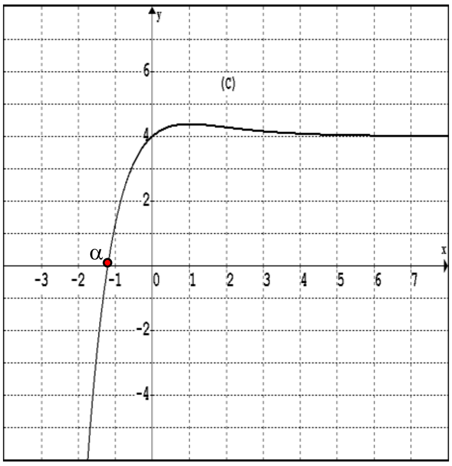
2) a- Determine a system of parametric equations of the line (D) that is perpendicular to (P) at A.
   b- Determine the coordinates of a point E on (D) such that $AE = \sqrt{5}$.

3) a- Show that the points B(0;–2;0) and C(2; 2;t) belong to (P). (t is a real number).
   b- Calculate t so that the triangle ABC is right at B and find in this case the volume of the tetrahedron EABC.

III- (8 points)

A-Consider the function f defined on IR by $f(x) = 4 + x\,e^{-x}$ whose representative curve (C) is shown in the adjacent figure.

(C) cuts the axis of abscissas in one point of abscissa α .



1) Use (C) to study the sign of f(x).

2) Use integration by parts to calculate $\int_0^2 x\,e^{-x}dx$, then calculate the area of the region bounded by the axis of ordinates, the axis of abscissas, the curve (C) and the straight line with equation x = 2 .

B- In all what follows, let $\alpha = -1.2$.

Consider the function g defined on IR, by $g(x) = 4x - 3 - (x+1)e^{-x}$ and designate by (G) its representative curve in an orthonormal system $(O\,;\vec{i}\,,\ \vec{j})$.

1) Verify that $\lim\limits_{x\to-\infty} g(x) = +\infty$ and determine g(-2.5) to the nearest 10^{-2}.

2) Calculate $\lim\limits_{x\to+\infty} g(x)$ and verify that the straight line (D) with equation y = 4x-3 is an asymptote of (G).

3) Determine the coordinates of A, the point of intersection of (G) with its asymptote (D), and study the position of (G) with respect to (D).

4) a- Verify that $g'(x) = f(x)$.

   b- Set up the table of variations of g.

5) Draw (D) and (G).