وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	دورة سنة 2009 العادية
مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة ساعتان	

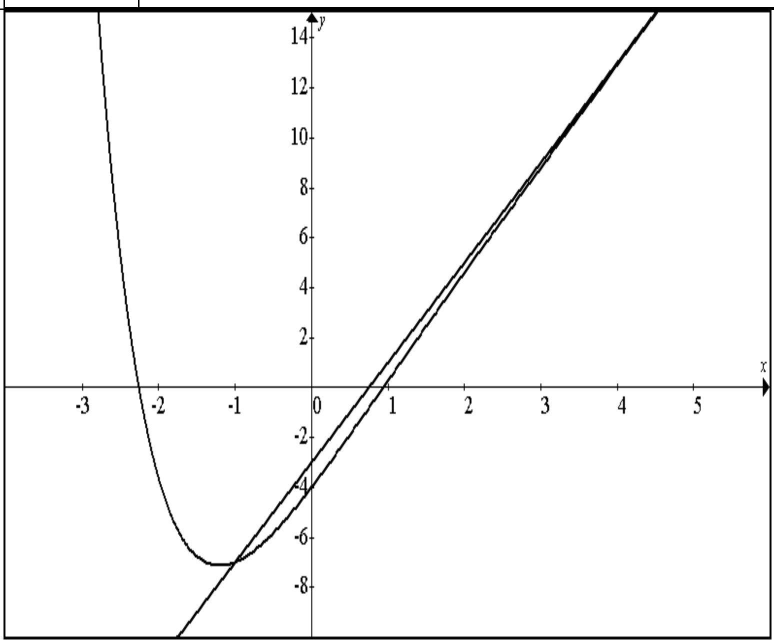
QI	Answer	M
1a	$1+i\sqrt{3}=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=2e^{i\frac{\pi}{3}}$;$2i=\ 2e^{i\frac{\pi}{2}}$; $z'=4e^{i\frac{5\pi}{6}}$.	0.5
1b	$\left\|\frac{z'}{z}\right\|=2$; $\arg\left(\frac{z'}{z}\right)=\frac{\pi}{3}[2\pi]$.	0.5
1c	OM=$\|z\|=2$; $OM'=\|z'\|=4$; $MM'=\|z'-z\|=2\sqrt{3}$ so $OM'^2=OM^2+MM'^2$ **OR** : $z'=(1+i\sqrt{3})2i=-2\sqrt{3}+2i$ M and M' have the same ordinate and M belongs to y-axis so OMM' is right at M.	1
2a	$(1+i)^3=2\sqrt{2}\ e^{i\frac{3\pi}{4}}$; $(1+i)^3=-2+2i$	0.5
2b	<u>Exponential form</u>: $z'=(1+i\sqrt{3})(1+i)^3=2e^{i\frac{\pi}{3}}\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^3=4\sqrt{2}e^{i\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)}=4\sqrt{2}e^{i\left(\frac{13\pi}{12}\right)}$. <u>Algebraic form:</u> $z'=\left(1+i\sqrt{3}\right)\left(1+i\right)^2\left(1+i\right)=\left(1+i\sqrt{3}\right)\left(2i-2\right)=2\left(-1-\sqrt{3}\right)+2\left(1-\sqrt{3}\right)i$	0.5
2c	Comparing the exponential and the algebraic forms of z': $4\sqrt{2}\cos\left(\frac{13\pi}{12}\right)=2\left(-1-\sqrt{3}\right)$, hence $\cos\frac{13\pi}{12}=\frac{-(\sqrt{2}+\sqrt{6})}{4}$	1

QII	Answer	M
A1a	$p(K)=\frac{2}{6}=\frac{1}{3}$; $p(L)=\frac{2}{3}$	0.5
A1b	$p(E\cap K)=p(K)\times p(E/K)=\frac{1}{3}\times\frac{C_1^1\times C_4^2}{C_5^3}=\frac{1}{5}$.	0.5
A1c	$p(E\cap L)=p(L)\times p(E/L)=\frac{2}{3}\times\frac{C_1^1\times C_4^1}{C_5^2}=\frac{4}{15}$. $p(E)=p(E\cap K)+p(E\cap L)=\frac{1}{5}+\frac{4}{15}=\frac{7}{15}$.	1
A2	$p(K/E)=\frac{p(E\cap K)}{p(E)}=\frac{\frac{1}{5}}{\frac{7}{15}}=\frac{3}{7}$.	0.5
B	The number of possible cases is $C_5^3=10$ $p(X=4)=p(0, 1, 4 \text{ or } 0, 2, 4 \text{ or } 1, 2, 4)=\frac{3}{10}$. <table><tr><td>$X=x_i$</td><td>2</td><td>4</td><td>5</td></tr><tr><td>p_i</td><td>$\frac{1}{10}$</td><td>$\frac{3}{10}$</td><td>$\frac{6}{10}$</td></tr></table>	1.5

QIII	Answer	M
1a	2-0-2=0 and 1+0-1=0.	0.5
1b	(d): x = m+1; y=2m; z = 5m+1	0.5
2a	$(D):\ x=2t+1$; $y=-t$; $z=1$	0.5
2b	$\overrightarrow{AE}(2t,-t,0)$; $AE=\sqrt{5t^2}=\sqrt{5}$, hence $t=\pm1$. For t = 1, E(3,-1,1).	0.5
3a	0+2-2=0; 4-2-2=0 then B and C belong to P.	0.5
3b	$\overrightarrow{AB}(-1,-2,-1)$; $\overrightarrow{BC}(2,4,t)$; $\overrightarrow{AB}.\overrightarrow{BC}=0$ so $t=-10$ Area of ABC = $\frac{\sqrt{6}\times\sqrt{120}}{2}=6\sqrt{5}$ Volume of EABC = $\frac{area(ABC)\times EA}{3}=\frac{6\sqrt{5}\times\sqrt{5}}{3}=10u^3$. ***OR*** Calculate the mixed product	1.5

<table>
<tr><th>QIV</th><th>Answer</th><th>M</th></tr>
<tr><td>A1</td><td>$f(x) = 0$ for $x = \alpha$; $f(x) > 0$ for $x > \alpha$; $f(x) < 0$ for $x < \alpha$</td><td>0.5</td></tr>
<tr><td>A2</td><td>Let $U = x; V' = e^{-x}$ then $U' = 1; V = -e^{-x}$ $\int_0^2 xe^{-x}dx = \left[-xe^{-x}-e^{-x}\right]_0^2 = -3e^{-2}+1$

Area = $\int_0^2 4dx + \int_0^2 xe^{-x}dx = \left[4x\right]_0^2 + \left[-xe^{-x}-e^{-x}\right]_0^2 = (-3e^{-2}+9)$ u^2</td><td>1.5</td></tr>
<tr><td>B1</td><td>$\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} \frac{4xe^x - 3e^x - x - 1}{e^x} = +\infty$ $\lim\limits_{x\to-\infty} 4xe^x = 0.$ and $\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} \frac{-x-1}{e^x}$

g(-2.5)=5.27.</td><td>1</td></tr>
<tr><td>B2</td><td>$\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty} (4x - 3 - xe^{-x} - e^{-x}) = +\infty$ $\lim\limits_{x\to+\infty} xe^{-x} = 0$
$\lim\limits_{x\to+\infty} (g(x) - (4x-3)) = \lim\limits_{x\to+\infty} -(x+1)e^{-x} = \lim\limits_{x\to+\infty} (-xe^{-x} - e^{-x}) = 0$
then the straight line with equation y = 4x - 3 is an asymptote of (G).</td><td>1</td></tr>
<tr><td>B3</td><td>$g(x) - (4x-3) = -(x+1)e^{-x}$
(G) intersects (D) at x = -1 thus A(-1;-7)
If $x < -1$, $-(x+1)e^{-x} > 0$, then (G) is above (D)
If $x > -1$, $-(x+1)e^{-x} < 0$, then (G) is below (D)</td><td>1</td></tr>
<tr><td>B4a</td><td>$g'(x) = 4 - e^{-x} + (x+1)e^{-x} = 4 + xe^{-x} = f(x)$</td><td>0.5</td></tr>
<tr><td>B4b</td><td><table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>-1.2</td><td></td><td>$+\infty$</td></tr>
<tr><td>g'(x)</td><td></td><td>-</td><td>0</td><td>+</td><td></td></tr>
<tr><td>g(x)</td><td>$+\infty$</td><td>$\searrow$</td><td>-7.1</td><td>$\nearrow$</td><td>$+\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td>B5</td><td></td><td>1.5</td></tr>
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