Third secondary class(LS)

Mathematics (last trial)

Exercise one:

In a complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider a point M , distinct from O , of affix z and appoint M' of affix z' such that $z' = \frac{z}{|z|^2}$.

1) Determine the algebraic form of z' in each of the following cases:

   $z = 2$; $z = \sqrt{2}e^{i\frac{\pi}{4}}$; $z = \sqrt{2}e^{-i\frac{\pi}{4}}$

2) Verify that : $z' = \frac{1}{\bar{z}}$.
3) Prove that the points O , M , M' are collinear.
4) Prove that $\overline{z' - 1} = \frac{1-z}{z}$.
5) In this part M describes a circle (C) of center E(1;0) and radius 1 , (M distinct from O)
   a- Verify that $|1 - z| = 1$
   b- Prove that $|z' - 1| = |z'|$ then deduce that M' describes a straight line (d) to be determined.

Exercise two:

In a farm , there are two sizes of rabbits , “small and big” , and 3 colors, “ white , brown and black” , are formed of 30 rabbits as shown in the following table.

Color Size	White	Brown	Black
Small	4	4	6
Big	10	2	4

Part A:

We select at random **one** rabbit out of **30** rabbits .

a- Calculate the probability for the chosen rabbit to be of brown color.

b- Calculate the probability for the chosen rabbit to be of black color from the small size.

c- Suppose that the selected rabbit is **not of brown color**.
Calculate then the probability for this rabbit to be from the big size.

Part B:

In this part , we select simultaneously and randomly **3** rabbits from the **24** rabbits that **doesnot have brown color**.

Let **X** be the random variable that is equal to the number of obtained rabbits that have white color and small size.

a-Verify that P(X=1) = $\frac{190}{506}$

b- Determine the probability distribution of **X**.

c-Calculate the expected value of X.

Exercise three:

In a space referred to a direct orthonormal system $(O;\vec{i};\vec{j};\vec{k})$, consider the plane (P) of equation $2x - y + z = 0$ and the plane (Q) of equation $x + y - z + 2\sqrt{3}$ =0 .

Let (d) be the line of intersection of (P) and (Q).

1) Prove that (P) and (Q) are perpendicular.
2) Write a system of parametric equations of line (d).
3) Let (C) be a circle , in plane (P) , of center O and radius 3.
   Prove that (d) cuts (C) in two points A and B.
4) Designate by E the midpoint of [AB] .
   Show that E is of coordinates ($\frac{-2\sqrt{3}}{3}$, $\frac{-2\sqrt{3}}{3}$,$\frac{2\sqrt{3}}{3}$) , then write a system of parametric equations of (OE).
5) Determine the equation of the mediator plane of [AB].

Exercise four:

The plane is referred to an orthonormal reference $(O;\vec{i},\vec{j})$.

Part A:

Consider the differential equation (E): $y' - y = x + 1$.

1) Determine the general solution of (E).
2) Determine the particular solution of (E) knowing that its representative curve passes through the origin of the reference.

Part B:

Let f be the defined by: $f(x) = 2e^{x} - x - 2$. Let (C) be its representative curve.

1)
   a- Study the variations of f.
   b- Prove that the straight-line (Δ) of equation $y = -x - 2$ is an asymptote to (C) at $-\infty$ and study the relative position of (C) and (Δ) then calculate $\lim\limits_{x\to+\infty} \frac{f(x)}{x}$.
2)
   a- Prove that the equation $f(x) = 0$ admits two real roots.
   b- Verify that 0 is one of these roots and that the second root α verifies the relations $2e^{\alpha} = \alpha + 2$ and $-1.6 < \alpha < -1.5$.
3)
   a- Write the equation of the tangent (d) to (C) at O
   b- Draw (Δ) , (d) and (C). **(Graphical unit: 2 cm)**
4)
   a- Calculate the area $A(\alpha)$ of the closed domain limited by (C) and the axis of abscissa.
   b- Prove that $A(\alpha) = -(2\alpha^{2} + 4\alpha)\text{cm}^{2}$.

GOOD WORK

Exercise one:

1) $z = 2 \to z' = \frac{1}{2}$ $z = \sqrt{2}e^{i\frac{\pi}{4}} \to z' = \frac{1}{2} + i\frac{1}{2}$ $z = \sqrt{2}e^{-i\frac{\pi}{4}} \to z' = \frac{1}{2} - i\frac{1}{2}$	1
2) $z' = \frac{z}{\lvert z\rvert^2} = \frac{z}{z\bar{z}} = \frac{1}{\bar{z}}$	0.5
3) $z' = \frac{1}{\bar{z}} \to arg(z') = -arg(\bar{z}) + 2k\pi$ $\left(\overrightarrow{OM}, \overrightarrow{OM'}\right) = 2k\pi \to O, M, M' arecollinear$	0.5
4) $Z' - 1 = \frac{1}{\bar{z}} - 1 = \frac{1-\bar{z}}{\bar{z}} \to \overline{Z' - 1} = \overline{\left(\frac{1-\bar{z}}{z}\right)} \to \overline{Z' - 1} = \frac{1-z}{z}$	0.5
5) a- $M \in C(O,1) \to \lvert z-1\rvert = 1 \to \lvert 1-z\rvert = 1$	0.5
b- $\lvert z'\rvert = \frac{1}{\lvert z\rvert}$ $and\ \lvert z'-1\rvert = \frac{1}{\lvert z\rvert}$ $\to \lvert z'\rvert = \lvert z'-1\rvert$ $\to OM' = EM'$ $\to (d)\ is\ the\ perpendicular\ bisector\ of\ [OE]$	1

Exercise two:

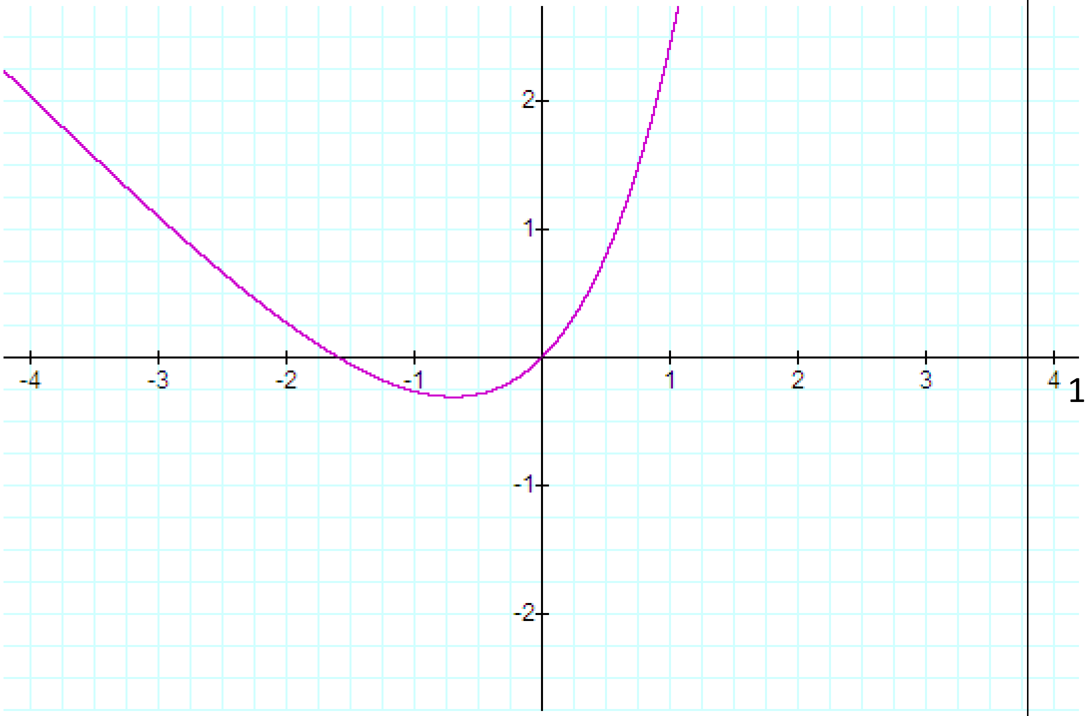
Part A:	
a- $P(brown\ color) = \frac{C_6^1}{C_{30}^1} = 0.2$	0.5
b- $P(black \cap small) = \frac{C_6^1}{C_{30}^1} = 0.2$	0.5
c- $P\left(big/\overline{brown}\right) = \frac{P(big \cap \overline{brown})}{P(\overline{brown})} = \frac{7}{12}$	0.75
Part B:	
a- $P(X=1) = \frac{C_4^1 C_{20}^2}{C_{24}^3} = \frac{190}{506}$	0.5
b- $X = \{0,1,2,3\}$ $P(X=0) = \frac{285}{506}$ $P(X=1) = \frac{190}{506}, P(X=2) = \frac{30}{506}, P(X=3) = \frac{1}{506}$	1
c- $E(X) = \sum p_i x_i = 0.5$	0.75

Exercise three:

1) $\overrightarrow{n_p} \cdot \overrightarrow{n_Q} = 0$	0.5
2) $(d): \begin{cases} x = \frac{-2\sqrt{3}}{3} \\ y = t - \frac{4\sqrt{3}}{3} \\ z = t \end{cases}$	0.75
3) $distance(O \to (d)) = 2 < 3 \to (C) cuts\ (d) at\ two\ points\ A\ and\ B$	1
4) $\overrightarrow{V_d} \cdot \overrightarrow{OE} = 0\ \ and\ \overrightarrow{OE} \cdot \overrightarrow{n_p} = 0\ then\ E\left(\frac{-2\sqrt{3}}{3}, \frac{-2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$ $(OE): \begin{cases} x = \frac{-2\sqrt{3}}{3} m \\ y = \frac{-2\sqrt{3}}{3} m \\ z = \frac{2\sqrt{3}}{3} m \end{cases}$	1.25
5) $Mediator\ plane\ of\ [AB]: y + z = 0$	0.5

Exercise four:

<table>
<tr><td>Part A:</td><td></td></tr>
<tr><td>1) $Y = ce^x - x - 2$</td><td>1</td></tr>
<tr><td>2) $Y(0) = 0 \to Y = 2e^x - x - 2$</td><td>0.5</td></tr>
<tr><td colspan="2">Part B:</td></tr>
<tr><td>1) a- $\lim_{x \to -\infty} f(x) = +\infty$ $\lim_{x \to +\infty} f(x) = +\infty$
$f'(x) = 2e^x - 1$
<table>
<tr><td>x</td><td>-∞</td><td></td><td>$-ln2$</td><td></td><td>+∞</td><td></td></tr>
<tr><td>$f'(x)$</td><td></td><td>$-$</td><td>0</td><td>$+$</td><td></td><td></td></tr>
<tr><td>$f(x)$</td><td>+∞</td><td>dec</td><td>$-1 + ln2$</td><td>inc</td><td></td><td>+∞</td></tr>
</table>
b- $\lim_{x \to -\infty} f(x) - (-x - 2) = 0 \to (\Delta) is\ an\ O.A\ to\ (C) at\ -\infty$
$f(x) - y_{(\Delta)} = 2e^x > 0\ for\ every\ x \in IR$</td><td>1.25</td></tr>
</table>

$\rightarrow (C)$ above (Δ) for every $x \epsilon IR$ $\text{Lim}_{x\rightarrow+\infty}\frac{f(x)}{x} = +\infty$ $\rightarrow (C)$ admits an asymptotical direction parallel to $y -$ axis at $+\infty$ 2) a- f is continuous , stictly decreasing and changes its sign over $]-\infty, -ln2[$ implies $f(x) = 0$ admits a uniqe solution over $]-\infty, -ln2[$ f is continuous , strictly increasing and changes its sign over $]-ln2, +\infty[$ implies $f(x) = 0$ admits a unique solution over $]-ln2, +\infty[$. Therefore $f(x) = 0$ admits two solutions over IR. b- $f(0) = 0$ $f(-1.6) \times f(-1.5) < 0 \rightarrow$ thesecond root of $f(x) = 0$ is α where $-1.6 < \propto < -1.5$ $f(\propto) = 0 \rightarrow 2e^{\alpha} = \alpha + 2$	1 0.75 1
3) a- $(d): y = x$  b-	0.5 1
4) a- $A(\propto) = \int_{\propto}^{0} -f(x)dx = -2 + 2e^{\propto} - \frac{1}{2}\propto^2 - 2\propto$ unit^2 b- $2e^{\alpha} = \propto + 2$ and unit $= 2cm \rightarrow A(\alpha) = -(2\alpha^2 + 4\alpha)cm^2$	0.5 0.5