ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the following points:
A (4 ;0; 1) , B(2 ;1 ;2), C(2 ; 0 ;3) and E(3 ; –1 ;0).

1) a- Write an equation of the plane (P) determined by A , B and C.
   b- Show that A is the orthogonal projection of E on (P).
2) a- Show that triangle ABC is right.
   b- Calculate the area of the triangle ABC.
   c- Calculate the volume of the tetrahedron EABC.
3) (Q) is the plane with equation $x - 2y - 2z - 2 = 0$.
   Show that (Q) passes through A and is perpendicular to (BE).
4) a- Write a system of parametric equations of the line (BC).
   b- Let M be a variable point on (BC). Prove that the distance from M to (Q) remains constant as M moves on (BC).

II- (4 points)

A shop sells two types of earphones E_1 and E_2 and three types of batteries B_1, B_2 and B_3.
During the promotion period, some items are placed in two baskets U and V.

Basket U contains 15 earphones of type E_1 and 5 earphones of type E_2;
Basket V contains 8 batteries of type B_1, 10 batteries of type B_2 and 7 batteries of type B_3.

A- A customer selects, at random, one item from each basket.

1) Show that the probability of obtaining an earphone E_1 and a battery B_1 is equal to $\frac{6}{25}$.
2) Calculate the probability that an earphone E_1 is among the two selected items.
3) The shop announces the following prices:

Item	Earphone E_1	Earphone E_2	Battery B_1	Battery B_2	Battery B_3
Price in LL	**40 000**	**15 000**	**30 000**	**25 000**	**50 000**

X is the random variable equal to the amount paid by the customer for buying the two selected items.

a- Prove that the probability P(X = 65 000) is equal to $\frac{37}{100}$.

b- Determine the probability distribution of X.

B- In this question, a customer selects, at random, an earphone from basket U and selects simultaneously and at random two batteries from basket V. Calculate the probability that the customer pays an amount less than or equal to 70 000LL.

III- (4 points)

The complex plane is referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$.

For every point M with affix z $(z \neq 0)$, we associate the point M' with affix z' such that $z' = \frac{2}{\bar{z}}$.

1) Let $z = re^{i\theta}$ $(r > 0)$, write z' in exponential form.
2) a- Show that $OM \times OM' = 2$.

   b- If $z = z'$, prove that M moves on a circle (C) whose center and radius are to be determined.
3) Let $z = 1 + iy$ where y is a real number.

   a- Prove that $|z' - 1| = 1$.

   b- As y varies, show that M' moves on a circle (C') whose center and radius are to be determined

IV- (8 points)

Consider the function f defined over $\mathbb{R}$ by $f(x) = (x+1)^2 e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-2)$.

   b- Determine $\lim\limits_{x \to +\infty} f(x)$ and deduce an asymptote to (C).
2) Show that $f'(x) = (1 - x^2)e^{-x}$ and set up the table of variations of f.
3) The line (d) with equation y = x intersects (C) at a point with abscissa α.

   Verify that $1.4 < \alpha < 1.5$.
4) Draw (d) and (C).

5) Let F be the function defined on $\mathbb{R}$ by $F(x) = (px^2 + qx + r) e^{-x}$.

   a- Calculate p, q and r so that F is an antiderivative of f .

   b- Calculate the area of the region bounded by (C), the axis of abscissas and the two lines with equations x = 0 and x = 1.

6) The function f has over $[1; +\infty[$ an inverse function h. Determine the domain of definition of h and draw its representative curve in the same system as (C).

I-	Solution	Mark
1a	For every point M(x; y ; z) of (P) ; $\vec{AM}.(\vec{AB}\wedge\vec{AC})=0$. So, (P): x + y + z –5 = 0 .	0.5
1b	$\vec{AE}(-1,-1,-1)$, $\vec{N_P}(1,1,1)$ then $\vec{AE}=-\vec{N_P}$. (AE) is perpendicular to (P) and $A\in(P)$. Thus A is the orthogonal projection of E on (P).	0.5
2a	$\vec{AB}(-2,1,1)$, $\vec{BC}(0,-1,1)$; $\vec{AB}.\vec{BC}=0$ Thus, ABC is right at B.	0.5
2b	Area (ABC) = $\frac{1}{2}AB\times BC=\frac{1}{2}\sqrt{6\times2}=\sqrt{3}\,u^2$	0.5
2c	$V=\frac{A_{(ABC)}\times AE}{3}=1u^3$. OR : By calculating the triple scalar product.	0.5
3	The coordinates of A satisfy the equation of (Q): 4 – 0 –2 –2 = 0 . $\vec{BE}(1,-2,-2)$ and $\vec{N_Q}$ (1 ; -2 ; -2) then (Q) passes through A and is per. to (BE)	0.5
4a	$\vec{BC}(0,-1,1)$, (BC): $x=2;\ y=-m+1;\ z=m+2\ ;m\in\mathbb{R}$.	0.5
4b	$d(M\to(Q))=\frac{\lvert 2+2m-2-2m-4-2\rvert}{\sqrt{1+4+4}}=2$.	0.5

<table>
<tr><th>II-</th><th>Solution</th><th>Mark</th></tr>
<tr><td>A1</td><td>$P(E_1 , B_1)=\frac{15}{20}\times\frac{8}{25}=\frac{120}{500}=\frac{6}{25}$.</td><td>0.5</td></tr>
<tr><td>A2</td><td>$P(E_1,B)=\frac{15}{20}\times1=\frac{3}{4}$.</td><td>0.5</td></tr>
<tr><td>A3a</td><td>$P(X=65000)=P(E_1,B_2)+P(E_2,B_3)=\frac{15}{20}\times\frac{10}{25}+\frac{5}{20}\times\frac{7}{25}=\frac{37}{100}$.</td><td>0.5</td></tr>
<tr><td>A3b</td><td><table>
<tr><td>$X=x_i$</td><td>40000</td><td>45000</td><td>65000</td><td>70000</td><td>90000</td></tr>
<tr><td>p_i</td><td>$\frac{1}{10}$</td><td>$\frac{2}{25}$</td><td>$\frac{37}{100}$</td><td>$\frac{6}{25}$</td><td>$\frac{21}{100}$</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>B</td><td>To pay a sum less than or equal to 70000LL, we cannot choose E_1 since 2 batteries cost at least 50000LL ; thus we choose $\{E_2,B_2,B_2\}$ or $\{E_2,B_1,B_2\}$
$P(S\le70000)=\frac{5}{20}\times\frac{C_{10}^2+C_8^1\times C_{10}^1}{C_{25}^2}=\frac{5}{48}$.</td><td>1</td></tr>
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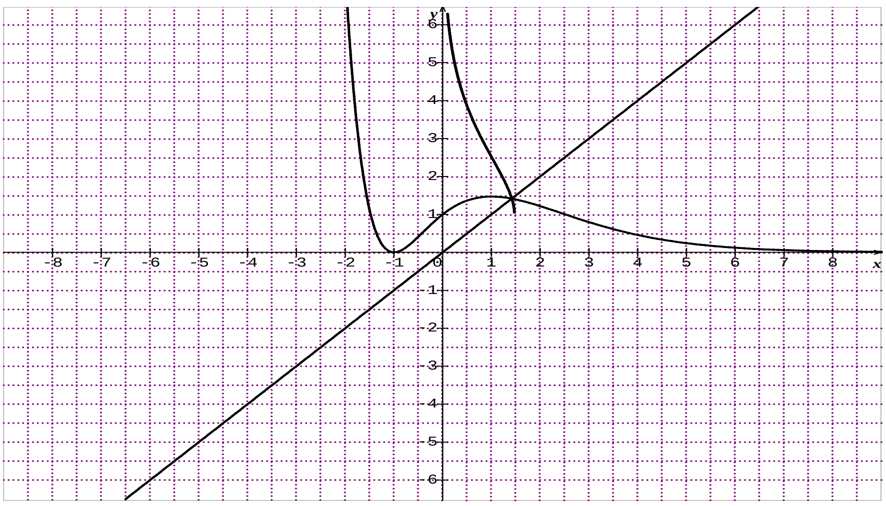
III	Solution	Mark
1	$z'=\frac{2}{re^{-i\theta}}=\frac{2}{r}e^{i\theta}$.	0.5
2a	$OM\times OM'=r\times\frac{2}{r}=2$. OR : $\lvert z'\rvert=\left\lvert\frac{2}{\bar{z}}\right\rvert=\frac{2}{\lvert z\rvert}$ hence $OM'=\frac{2}{OM}$.	0.5
2b	If z = z ' then $OM^2=2$; $OM=\sqrt{2}$. M moves on a circle with center O and radius $\sqrt{2}$.	1
3a	$\lvert z'-1\rvert=\left\lvert\frac{2}{1-iy}-1\right\rvert=\left\lvert\frac{1+iy}{1-iy}\right\rvert=\frac{\sqrt{1+y^2}}{\sqrt{1+y^2}}=1$.	1
3b	Let I be the point with affix 1. IM' = 1. Thus, M' moves on the circle (C') with center I(1 ; 0) and radius 1.	1

IV	Solution	Mark
1a	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} (x+1)^2 e^{-x} = +\infty$; f(– 2) = 7.4 .	0.5
1b	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{(x+1)^2}{e^x} = \lim\limits_{x\to+\infty} \frac{2(x+1)}{e^x} = \lim\limits_{x\to+\infty} \frac{2}{e^x} = 0.$ The x-axis is an asymptote to (C).	0.5
2	$f'(x) = 2(x+1)e^{-x} - e^{-x}(x+1)^2 = (1-x^2)e^{-x}$ $\begin{array}{c\|ccccccc} x & -\infty & & -1 & & 1 & & +\infty \\ \hline f'(x) & & - & 0 & + & 0 & - & \\ \hline f(x) & +\infty & \searrow & 0 & \nearrow & \frac{4}{e} & \searrow & 0 \end{array}$	1.5
3	f(1.4) = 1.42 > 1.4 ; f(1.5) = 1.39 < 1.5 thus1.4 < α < 1.5.	1
4		1.5
5a	F'(x) = f(x) so $-p x^2 + (2p - q)x + q - r = x^2 + 2x + 1$ for all real numbers x. Hence, p = – 1 , q = – 4 , r = – 5.	1
5b	Area= $\int_0^1 f(x)\,dx = \left[(-x^2 - 4x - 5)e^{-x}\right]_0^1 = (5 - \frac{10}{e}) = 1.321\ u^2$.	1
6	$D_h = \left]0;\frac{4}{e}\right]$, (C_h) is symmetric to (C) with respect to the straight line with equation y = x.	1