عدد المسائل : اربع	مسابقة في مادة الرياضيات	الاسم:
	المدة ساعتان	الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I-(4 points)

The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$.

Consider the plane (P) with equation $x-2y+2z-6=0$ and the two lines (d) and (d') defined as:

$$(d):\begin{cases}x=m+1\\ y=2m+1\\ z=2m+2\end{cases}\quad \text{and}\quad (d'):\begin{cases}x=2t\\ y=5t-3\\ z=4t\end{cases}\qquad (m \text{ and } t \text{ are real parameters})$$

1) Find the coordinates of A, the intersection point of line (d) and plane (P).

2) Verify that A is on line (d'), and that (d') is contained in plane (P).

3) a- Write an equation of plane (Q) determined by the lines (d) and (d').

b- Show that the two planes (P) and (Q) are perpendicular.

4) Let $B(1;1;2)$ be a point on (d).

Calculate the distance from point B to line (d').

II- (4 points)

The complex plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$.

Consider the points A (– i), B (– 2) and M (z) where z is a complex number different from -2.

Let M' be the point with affix z' such that $z'=\dfrac{1-iz}{z+2}$.

1) a- Find the algebraic form of the complex number $(z'+i)(z+2)$.

b- Give a geometric interpretation to $|z'+i|$ and $|z+2|$, then deduce that $AM'\times BM=\sqrt{5}$.

c- As M moves on the circle with center B and radius 1, show that M' moves on a circle whose center and radius are to be determined.

2) Suppose that $z=-2+iy$ with y a nonzero real number.

a- Find in terms of y the algebraic form of z'.

b- Determine the point M for which z' is real.

III-(4 points)

Consider two urns A and B.

- Urn Acontains two balls numbered 1 and 2.
- Urn B contains four balls numbered 1, 2, 3 and 4.

1)One of the two urns A and B is randomly chosen, after which a ball is randomly selected from thisurn.

Consider the following events:

- A: « the chosen urn is A»;
- N: « the selected ball is numbered 1».

a- Calculate the probabilities$P(N/A)$ and $P(N\cap A)$.

b- Show that $P(N)=\frac{3}{8}$ and deduce$P(A/N)$.

2) In this part, the six balls from the two urns A and B are placed in one urn W.
Two balls are selected randomly and simultaneouslyfrom the urn W.
Consider the following events:

- E: « the two selected balls carry the same number »;
- F: « the sum of numbers carried by the two selected balls is odd ».

a- Verify that $P(E)=\frac{2}{15}$.

b- Calculate P(F) and $P(F/E)$

IV- (8 points)

A-

Let g be the function defined on IRas $g(x)=x-1+e^{x}$.

1) Show that g is strictly increasing onIR. Set up the table of variations of g.

2) Calculate $g(0)$, then study according to the values of x the sign of $g(x)$.

B-

Let f be the function defined on IR as $f(x)=\frac{(x-2)e^{x}}{1+e^{x}}$ and (C) its representative curve in an orthonormalsystem $(O;\vec{i},\vec{j})$. Denote by (Δ) the line with equation $y=x-2$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$. Deduce an asymptote to (C).

2) Study, according to the values of x, the relative positions of (C) and (Δ).

3) Determine $\lim\limits_{x\to+\infty} f(x)$ and show that (Δ) is anasymptote to (C).

4) Showthat $f'(x)=\frac{e^{x}g(x)}{(1+e^{x})^{2}}$, then set up the table of variations of f.

5) Plot (Δ) and (C).

6) The function f has over $[0;+\infty[$ an inverse function h. Calculate $h'(0)$.

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة العام 2015 العادية السبت 13 حزيران 2015

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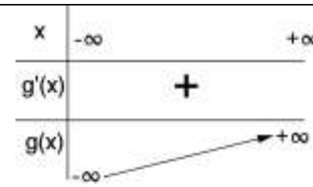
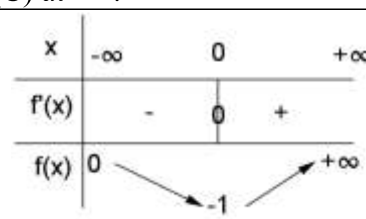
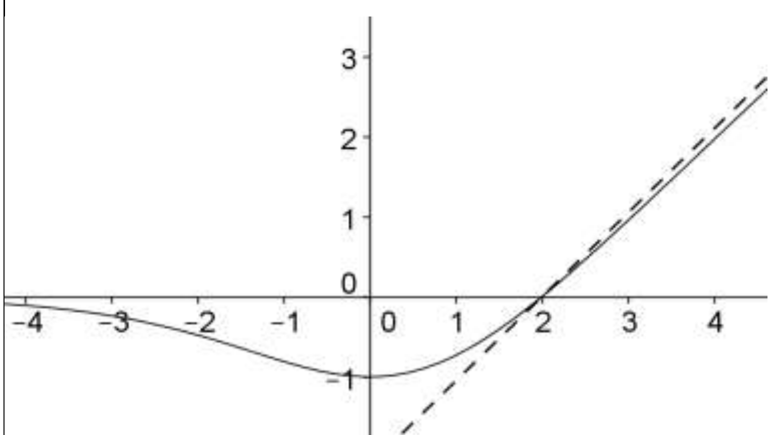
Q1	Answers	M
1	m + 1 – 4m- 2+4m - 6 = 0; m = 3 then A(4;7;8)	1
2	4=2t ; 7 = 5t – 3; 8 = 4t thus t = 2 unique value,hence, A belongs to (d') 2t – 10t +6 +8t -6 =0. So (d') is included in (P).	½
3a	$\overrightarrow{AM}.\left(\vec{V}\wedge\vec{V'}\right)=0$; $\begin{vmatrix} x-4 & y-7 & z-8 \\ 1 & 2 & 2 \\ 2 & 5 & 4 \end{vmatrix}=0$; $2x-z=0:(Q)$	1
3b	$\vec{N}.\vec{N'}=2+0-2=0$; $(P)\perp(Q)$	½
3c	$B\in(d)$ and $(P)\perp(Q)$ and as (d') is the intersection line of the two planes (P) and (Q) hense: $d(B;(d'))=d(B;(P))=\frac{\lvert x_B-2y_B+2z_B-6\rvert}{\sqrt{1+4+4}}=\frac{3}{3}=1$	1

Q2	Answers	M
1a	$z'+i=\frac{1-iz+iz+2i}{z+2}=\frac{1+2i}{z+2}$ hence $(z'+i)(z+2)=1+2i.$	1
1b	$\lvert z'+i\rvert=AM';\lvert z+2\rvert=BM$ $AM'\times BM=\lvert z'+i\rvert\times\lvert z+2\rvert=\lvert 1+2i\rvert=\sqrt{5}.$	1
1c	$M\in C(B;1)$; $BM=1$; $AM'=\sqrt{5}$ and M' moves on the circle with center A and radius $\sqrt{5}$.	½
2a	$z=-2+iy; z'=\frac{2}{y}+i\frac{-y-1}{y}$	1
2b	z' is real ; Im(z')=0 ;-y-1=0 then $M(-2;-1)$	½

Q3		Answers	M
1	a	P (N/A)= $\frac{1}{2}$; $P(N\cap A)=P(A)\times P(N/A)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.	1
	b	P(N) = $P(N\cap A)+P(N\cap\overline{A})=\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$; P(A/N)= $\frac{P(A\cap N)}{P(N)}=\frac{1/4}{3/8}=\frac{2}{3}$	1
2	a	P(E) $=\frac{C_2^2}{C_6^2}+\frac{C_2^2}{C_6^2}=\frac{2}{15}$.	½
	b	The outcomes of F are : (1 and 2) or (1 and 4) or (2 and 3) or (3 and 4), $P(F)=\frac{C_2^1\times C_2^1+C_2^1\times C_1^1+C_2^1\times C_1^1+C_1^1\times C_1^1}{C_6^2}=\frac{3}{5}$ and $P\left(F/\overline{E}\right)=\frac{P(F\cap\overline{E})}{p\left(\overline{E}\right)}=\frac{P(F)}{1-p(E)}=\frac{9}{13}$.	1 ½

Q4		Answers	M
A	1	$g'(x) = 1 + e^x > 0$ for all x hence g is strictly increasing over IR . 	1
	2	g is strictly increasing over IR and g (0) = 0. If x< 0 then g(x) < 0. for x > 0 then g(x) > 0.	1
B	1	$\lim\limits_{x\to-\infty} f(x) = \dfrac{\lim\limits_{x\to-\infty}(x-2)e^x}{\lim\limits_{x\to-\infty}(1+e^x)} = 0.$ The x-axis is an asymptote to (C) at -∞	1
	2	f (x) –(x–2) = $\dfrac{-x+2}{1+e^x}$. Hence, (C) is above (d) for x <2, (C) is below (d) for x >2 and (C) intersects (d) at x=2.	1
	3	$\lim\limits_{x\to+\infty} f(x) = +\infty$ $\lim\limits_{x\to-\infty}[f(x)-(x-2)] = \lim\limits_{x\to+\infty}\dfrac{-x}{e^x} = 0.$ So the line with equation y = x-2 is an asymptote to (C) at +∞.	1
	4	$f'(x) = \dfrac{\left[e^x + e^x(x-2)\right]\left[1+e^x\right] - e^x(x-2)e^x}{(1+e^x)^2}$ $= \dfrac{e^x(x-1+e^x)}{(1+e^x)^2} = \dfrac{e^x g(x)}{(1+e^x)^2}.$ 	1
	5		1
	6	Since f(2) = 0 henceh'(0)= $\dfrac{1}{f'(2)} = \dfrac{1+e^2}{e^2}$	1