وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠١٥ الاستثنائيّة السبت ٢٢ آب ٢٠١٥
عدد المسائل : اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I-(4 points)

The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$.

Consider the plane (P) with equation $x-2y+2z-6=0$ and

the line (d) with parametric equations $\begin{cases} x = m+1 \\ y = 2m+1 \\ z = 2m+2 \end{cases}$ $(m\in\mathbb{R})$.

Let (Q) be the plane containing (d) and perpendicular to (P) and A (1; 1; 2) a point on (d).

1) Show that $2x-z=0$ is an equation of the plane (Q).

2) Prove that the line (Δ) with parametric equations $\begin{cases} x = 2t \\ y = 5t-3 \\ z = 4t \end{cases}$ $(t\in\mathbb{R})$

is the line of intersection of (P) and (Q).

3) a- Determine the coordinates of B, the meeting point of (d) and (Δ).
b- Determine the coordinates of point F, the orthogonal projection of A on (Δ).
c- Calculate the cosine of the angle formed by (d) and (P).

II-(4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points

A, B, M and M´ with respective affixes -2 ; i ; z and z' so that $z'=\dfrac{z+2}{z-i}$ with $z\neq -2$ and $z\neq i$.

1) In this part only, assume that $z=\sqrt{2}e^{i\frac{3\pi}{4}}$.
a- Write the complex number $-1-i$ in exponential form.
b- Deduce that $(z')^{40}$ is a real number.

2) Let $z=x+iy$ and $z'=x'+iy'$ where x, y, x' and y' are real numbers.
a- Calculate x' and y' in terms of x and y.
b- Express the scalar product $\overrightarrow{AM}.\overrightarrow{BM}$ in terms of x and y.
c- Deduce that if z' is pure imaginary, then the two lines (AM) and (BM) are perpendicular.

3) a- Verify that $(z'-1)(z-i)=2+i$.
b- Deduce that if M moves on the circle (C) with center B and radius $\sqrt{5}$, then M' moves on a circle (C') with center and radius to be determined .

III-(4 points)

An urn contains 4 red balls and 3 black balls.

A- In this part, we select randomly and one after another, three balls from this urn as follows: we select the first ball without putting it back in the urn, then we select the second ball and we put it back in the urn and finally, we select the third ball.

1) Verify that the probability to select three black balls is $\frac{1}{21}$.
2) Calculate the probability that the first ball selected is black and the two other balls are red.
3) Knowing that the first ball selected is black, calculate the probability that the two other balls are red.

B- In this part, the red balls are numbered: 1; 2; 3; 4 and the black balls are numbered: 1; 2; 3. We select randomly and **simultaneously** three balls from the urn.

1) Calculate the probability that two balls among the three balls selected have the same number.
2) Let X be the random variable that is equal to the number of selected balls which are numbered 2. Determine the probability distribution of X.

IV-(8 points)

A- Let g be the function defined on $]0;+\infty[$ as $g(x) = x^3 - 1 + 2\ln x$.

1) Determine $\lim_{x\to 0} g(x)$ and $\lim_{x\to +\infty} g(x)$.
2) Calculate $g'(x)$ then set up the table of variations of g.
3) Calculate $g(1)$ then deduce the sign of g(x) according to the value of x.

B- Consider the function f defined on $]0;+\infty[$ as $f(x) = x - \frac{\ln x}{x^2}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. Let (d) be the line with equation $y = x$.

1) Determine $\lim_{x\to 0} f(x)$ and deduce an asymptote to (C).
2) a- Discuss, according to the values of x, the relative positions of (C) and (d).
   b- Determine $\lim_{x\to +\infty} f(x)$ and show that (d) is an asymptote to (C).
3) a- Verify that $f'(x) = \frac{g(x)}{x^3}$ and set up the table of variations of f.
   b- Determine the point E on (C) where the tangent (Δ) to (C) is parallel to (d).
   c- Plot (d), (Δ) and (C).
4) Let α be a real number greater than 1. Denote by $A(\alpha)$ the area of the region bounded by (C), (d) and the two lines with equations x = 1 and $x = \alpha$.

   a- Verify that $\int \frac{\ln x}{x^2}dx = \frac{-1-\ln x}{x} + k$, where k is a real number.

   b- Express $A(\alpha)$ in terms of α.

   c- Using the graphic, show that $A(\alpha) < \frac{(\alpha-1)^2}{2}$.

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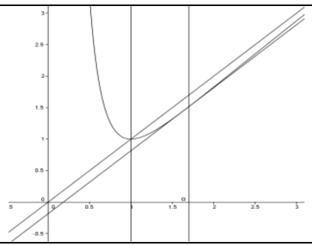
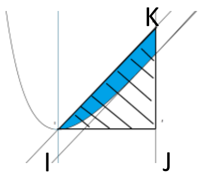
Q.I	Answers	N
1	2(m + 1) – (2 m+2) = 0, $\overrightarrow{N}\cdot\overrightarrow{N_P} = 2-2=0$ hence 2x – z = 0 is an equation of (Q). **OR**: $\overrightarrow{AM}\cdot\left(\overrightarrow{V_d}\wedge\overrightarrow{N_P}\right)=0 \Leftrightarrow 2x-z=0.$	1
2	2t – 2t = 0 ; t –5t + 6 + 4t – 6 = 0 hence (Δ) verifies the equations of the two planes. **OR**: we solve the system $\begin{cases}2x-z=0\\x-2y+2z-6=0\end{cases}$ taking x = t.	0,5
3.a	B is the point of intersection of (d) and (P): m + 1 –4m –2 +4m + 4 – 6 = 0 $\rightarrow$ m = 3. B (4 ; 7 ; 8). **OR**: we solve the system $\begin{cases}m+1=2t\\2m+1=5t-3\\2m+2=4t\end{cases}$	1
3.b	If F is the orthogonal projection of A on (Δ) then $\overrightarrow{AF}\cdot\overrightarrow{V_\Delta}=0$.; $\overrightarrow{V_\Delta}(2;5;4)$ and $\overrightarrow{AF}(2t-1;5t-4;4t-2)$; $4t-2+25t-20+16t-8=0$ then $t=\frac{2}{3}\rightarrow F\left(\frac{4}{3};\frac{1}{3};\frac{8}{3}\right)$.	1
3.c	If α is a measure of the angle of (d) and (P) then $\cos\alpha=\frac{BF}{AB}=\frac{4\sqrt{5}}{9}$.	0,5

Q.II	Answers	N
1.a	$-1-i=\sqrt{2}e^{i\frac{5\pi}{4}}$	0.5
1.b	$z=\sqrt{2}e^{i\frac{3\pi}{4}}=\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=-1+i.\ z'=\frac{-1+i+2}{-1+i-i}=-1-i=\sqrt{2}e^{i\frac{5\pi}{4}}$. $(z')^{40}=2^{20}e^{50\pi i}=2^{20}e^{0\pi i}=2^{20}$ then $(z')^{40}$ is a real number	0.5
2.a	$z'=\frac{z+2}{z-i}=\frac{x+2+iy}{x+i(y-1)}\times\frac{x-i(y-1)}{x-i(y-1)}=\frac{x^2+y^2+2x-y}{x^2+(y-1)^2}+i\frac{x-2y+2}{x^2+(y-1)^2}$. then $x'=\frac{x^2+y^2+2x-y}{x^2+(y-1)^2}$ et $y'=\frac{x-2y+2}{x^2+(y-1)^2}$.	1
2.b	$\overrightarrow{AM}.\overrightarrow{BM}=x^2+y^2+2x-y$.	0.5
2.c	$\overrightarrow{AM}.\overrightarrow{BM}=x^2+y^2+2x-y$. If z' is pure imaginary so $\frac{x^2+y^2+2x-y}{x^2+(y-1)^2}=0$; $x^2+y^2+2x-y=0$; $\overrightarrow{AM}.\overrightarrow{BM}=0$ Therefore (AM) and (BM) are perpendicular.	0.5
3.a	$(z'-1)(z-i)=\left(\frac{z+2}{z-i}-1\right)(z-i)=\frac{2+i}{z-i}(z-i)=2+i.$	0.5
3.b	If M moves on the circle with center B and radius $\sqrt{5}$ so $\|z-i\|=\sqrt{5}$; $\|z'-1\|\|z-i\|=\|2+i\|=\sqrt{5}$; $\|z'-1\|=\frac{\sqrt{5}}{\|z-i\|}=\frac{\sqrt{5}}{\sqrt{5}}=1$ therefore M′ moves on the circle with center H(1) and radius1.	0.5

Q.III		Answers			N
A	1	P (N, N, N) = $\frac{3}{7}\times\frac{2}{6}\times\frac{2}{6}=\frac{1}{21}$.	0.5 ⇨ 2	P (N, R, R) = $\frac{3}{7}\times\frac{4}{6}\times\frac{4}{6}=\frac{4}{21}$.	0.5
	3	P (the two other balls are red / the first ball selected is black) = $\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$.			0.5
B	1	P(two balls having the same number) = P{1,1,x}+P(2,2,x}+P{3,3,x}= $3\times\frac{C_2^2\times C_5^1}{C_7^3}=\frac{15}{35}=\frac{3}{7}$.			1
	2	$X=\{0;1;2\}$ $P(X=0)=\frac{C_5^3}{C_7^3}=\frac{10}{35}=\frac{2}{7}$; $P(X=1)=\frac{C_2^1\times C_5^2}{C_7^3}=\frac{20}{35}=\frac{4}{7}$; $P(X=2)=\frac{C_2^2\times C_5^1}{C_7^3}=\frac{5}{35}=\frac{1}{7}$			1.5

Q.IV		Answers	N
A	1	$\lim_{x\to 0} g(x)=-\infty$ and $\lim_{x\to+\infty} g(x)=+\infty$.	0,5
	2	$g'(x)=3x^2+\frac{2}{x}>0$. x: 0, +∞ g´(x): + g(x): −∞ ↗ +∞	0,5
	3	g is strictly increasing on $]0;+\infty[$ and g(1) = 0 then : if $0<x\le 1$ then $g(x)\le g(1)$ and $g(x)\le 0$. if $x>1$ then $g(x)>g(1)$ and $g(x)>0$.	0,5
B	1	$\lim_{x\to 0} f(x)=+\infty$; the line of equation x=0 is an asymptote to (C).	0,5
	2.a	$f(x)-y=-\frac{\ln x}{x^2}$. (C) and (d) intersect at the point (1; 1). For $0<x<1$; $f(x)-y>0$; (C) is above (d) ; For $x>1$; $f(x)-y<0$, (C) is below (d).	0,5
	2.b	$\lim_{x\to+\infty}\frac{\ln x}{x^2}=\lim_{x\to+\infty}\frac{\frac{1}{x}}{2x}=\lim_{x\to+\infty}\frac{1}{2x^2}=0 \Rightarrow \lim_{x\to+\infty} f(x)=+\infty$; $\lim_{x\to+\infty}[f(x)-y]=\lim_{x\to+\infty}-\frac{\ln x}{x^2}=0$; y = x asymptote to (C).	¾
	3.a	$f'(x)=1-\frac{x-2x\ln x}{x^4}=1-\frac{1-2\ln x}{x^3}$ $=\frac{x^3-1+2\ln x}{x^3}=\frac{g(x)}{x^3}$. x: 0, 1, +oo f´(x): −, 0, + f(x): + oo ↘ 1 ↗ +oo	1
	3.b	$f'(x)=1; g(x)=x^3$; $-1+2\ln x=0$; $x=e^{\frac{1}{2}}$; $f\left(e^{\frac{1}{2}}\right)=e^{\frac{1}{2}}-\frac{1}{2e}$. $E\left(e^{\frac{1}{2}};e^{\frac{1}{2}}-\frac{1}{2e}\right)$.	¾
	3.c	 1 ⇨ 4.a $u=\ln x$ and $v'=\frac{1}{x^2}$; $u'=\frac{1}{x}$ and $v=-\frac{1}{x}$. $\int\frac{\ln x}{x^2}dx=\frac{-\ln x}{x}+\int\frac{1}{x^2}dx=\frac{-\ln x}{x}-\frac{1}{x}+k=\frac{-1-\ln x}{x}+k$	1
	4.b	$A(\alpha)=\int_1^{\alpha}[x-f(x)]dx=\int_1^{\alpha}\frac{\ln x}{x^2}dx=\left[-\frac{\ln x}{x}\right]_1^{\alpha}-\left[\frac{1}{x}\right]_1^{\alpha}=(1-\frac{1}{\alpha}-\frac{\ln\alpha}{\alpha})\ u^2$.	0,5
	4.c	 $A(\alpha)<\int_1^{\alpha}(x-1)dx$; $A(\alpha)<\frac{\alpha^2}{2}-\alpha+\frac{1}{2}$; $A(\alpha)<\frac{(\alpha-1)^2}{2}$ since the colored region is inside the right isosceles triangle IJK **OR** $A(\alpha)<$ Area of the triangle IJK; $A(\alpha)<\frac{IJ\times JK}{2}$; $A(\alpha)<\frac{(\alpha-1)^2}{2}$.	0,5