وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسميّة

امتحانات الشهادة الثانوية العامة
الفرع : علوم الحياة

دورة العام ٢٠١٦ الاستثنائيّة
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عدد المسائل: أربع

مسابقة في مادة الرياضيات
المدة ساعتان

الاسم:
الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the complex plane referred to an orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A(1), M(z) and M'(z') so that: $z'=\left(1-i\right)z+i$ with $z\neq 1$.

1) a- Verify that $z'-1=\left(1-i\right)\left(z-1\right)$.

b- Verify that $AM'=AM\sqrt{2}$. Deduce that if M moves on the circle with center A and radius $\sqrt{2}$, then M' moves on a circle (C) whose center and radius should be determined.

c- Prove that: $\left(\vec{u};\overrightarrow{AM'}\right)=-\frac{\pi}{4}+\left(\vec{u};\overrightarrow{AM}\right)+2k\pi$ with $k\in\mathbb{Z}$.

d- Compare $\left|z'-z\right|$ and $\left|z-1\right|$, then prove that the triangle AMM' is right isosceles.

2) Let $z=x+iy$ and $z'=x'+iy'$ where $x,\ y,\ x'$ and y' are real numbers.

a- Express x' and y' in terms of x and y.

b- Verify that if M' moves on a line (D) with equation $y=x$, then M moves on a line (Δ) to be determined.

II- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider:

- The plane (P): $x-2y+z=0$; and the plane (Q) : $x+y+z+3=0$
- The two points $A(1;0;-1)$ and $E(0;-1;-2)$
- In the plane (P), the circle (C) with center A and radius $R=\sqrt{3}$.

Let (Δ) be the intersection line of (P) and (Q).

1) a- Prove that (P) is perpendicular to (Q).

b- Verify that the line (Δ) is defined by: $x=-t-2$; $y=-1$; $z=t$ ($t\in\mathbb{R}$).

c- Prove that E is the orthogonal projection of A on (Δ) .

d- Deduce that (Δ) is tangent to (C) at E.

2) Denote by H the point on (Δ) with positive abscissa so that $EH=3\sqrt{2}$. Determine the coordinates of H.

3) Let (T) be the second tangent through H to (C). Denote by F the point of tangency between (T) and (C).

Determine a system of parametric equations of one bisector of the angle EHF.

III- (4 points)

Consider an urn U containing three dice:

- **Two** red dice where the faces of each of them are numbered from 1 to 6
- **One** black die where **two** of its faces are numbered 6 and the **four** others are numbered 1.

A player selects randomly and simultaneously two dice from the urn, then he rolls them only once.
Consider the following events:

A : «The two dice selected are red».

$\overline{A}$: «The two dice selected are one red and one black».

L : «Out of the two dice, only one shows the number 6».

1) Calculate the probability P(A).
2) a- Verify that $P(L/A) = \frac{5}{18}$ and calculate $P(A \cap L)$.

   b- Calculate $P(\overline{A} \cap L)$ and verify that $P(L) = \frac{19}{54}$.
3) Knowing that only one of the two dice shows the number 6, calculate the probability that the two dice selected are red.
4) Calculate the probability that at least one die shows the number 6.

IV- (8 points)

Let f be the function defined over $\mathbb{R}$ as $f(x) = x + xe^{-x}$, and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Calculate $f(-1.5)$.
2) Let (d) be the line with equation $y = x$.

   a- Discuss according to the values of x, the relative position of (C) and (d).

   b- Prove that (d) is an asymptote to (C).
3) A is the point on (C) where the tangent (T) to (C) is parallel to (d). Determine the coordinates of A and write an equation of (T).
4) The following table is the table of variations of the function f ', the derivative of f .

x	$-\infty$		2		$+\infty$
f ''(x)		−	0	+	
f '(x)	$+\infty$	↘	$1-e^{-2}$	↗	1

   a- Verify that (C) admits an inflection point W whose coordinates should be determined.

   b- Verify that f is strictly increasing over $\mathbb{R}$, then set up the table of variations of the function f.
5) Draw (d), (T) and (C).
6) a- Prove that f has an inverse function h whose domain of definition should be determined.

   b- Draw the curve (C') of h in the same system as that of (C).
7) Let M be any point on (C) with abscissa $x \geq 0$, and denote by N the symmetric of M with respect to (d).

   a- Calculate MN in terms of x.

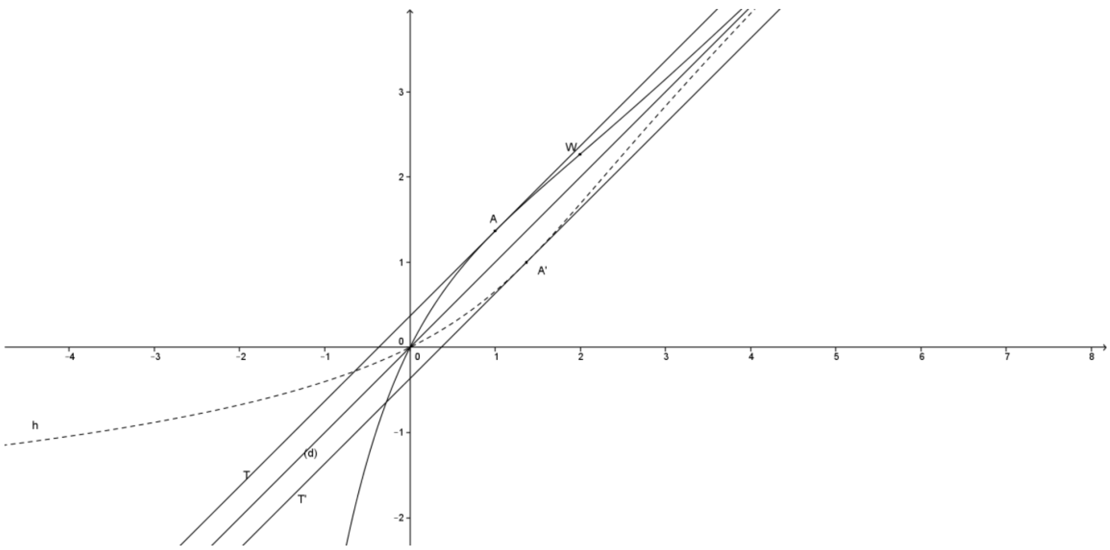
   b- Calculate the maximum value of MN.

مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة: ساعتان

I	Solution	M
1a	$z'-1=(1-i)z+i-1$; $z'=(1-i)(z-1)$	0.25
1b	$\lvert z'-1\rvert=\lvert(1-i)(z-1)\rvert=\lvert 1-i\rvert\times\lvert z-1\rvert$ $\lvert z_{M'}-z_A\rvert=\sqrt{2}\lvert z_M-z_A\rvert; AM'=\sqrt{2}AM$ $AM=\sqrt{2}$; $AM'=2$; $M'\in C(A;2)$	1
1c	$\arg[z'-1]=\arg[(1-i)(z-1)]=\arg(1-i)+\arg(z-1);$ $\arg(\vec{u};\overrightarrow{AM'})=-\frac{\pi}{4}+(\vec{u};\overrightarrow{AM})+2k\pi$	1
1d	$\lvert z'-z\rvert=\lvert(1-i)z+i-z\rvert=\lvert -iz+i\rvert=\lvert i\rvert\lvert z-1\rvert=\lvert z-1\rvert;$ $\arg(z'-z)=\arg[-i(z-1)]$ $\arg(\vec{u};\overrightarrow{AM'})=-\frac{\pi}{4}+(\vec{u};\overrightarrow{AM})$ then $MAM'=45^\circ$ et $\lvert z'-z\rvert=\lvert z-1\rvert; MM'=AM$ Then AMM' is right isosceles at M.	0.75
2a	$x'=x+y$; $y'=y-x+1$	0.5
2b	$x'=y'$; then M moves on the line $(\Delta): x=\frac{1}{2}$	0.5

II	Solution	M
1a	$\overrightarrow{n_{(P)}}(1;-2;1)$ and $\overrightarrow{n_{(P)}}(1;1;1)$; $\overrightarrow{n_{(P)}}.\overrightarrow{n_{(P)}}=0$	0.5
1b	$-t-2+2+t=0$ then $(\Delta)\subset(P)$; $-t-2-1+t+3=0$ then $(\Delta)\subset(Q)$.	0.5
1c	$\overrightarrow{AE}(-1;-1;-1);\overrightarrow{AE}.\overrightarrow{V_{(\Delta)}}=0$ and for $t=-2$ $E\in(\Delta)$ so E is the projection of A on (Δ)	0.5
1d	$AE=\sqrt{3}=R;(\Delta)$ is perpendicular to (AE) at E then (Δ) is the tangent to (C) at E.	0.5
2	$H(-t-2;-1;t); EH=3\sqrt{2}; EH=\sqrt{2(t+2)^2}$; $t=1$ not accepted. $t=-5$ accepted $\Rightarrow H(3;-1;-5)$	1
3	$(AH):\begin{cases}x=2m+1\\y=-m\\z=-4m-1\end{cases}$	1

III	Solution	M
1	$p(A)=\frac{C_2^2}{C_3^2}=\frac{1}{3}$	0.5
2a	$p(L/A)=\frac{1}{6}\times\frac{5}{6}\times 2=\frac{5}{18}$; $P(A\cap L)=p(L/A)\times p(A)=\frac{5}{18}\times\frac{1}{3}=\frac{5}{54}$	1
2b	$p(\overline{A}\cap L)=p(L/\overline{A})\times p(\overline{A})=\frac{2}{3}(\frac{1}{6}\times\frac{4}{6}+\frac{5}{6}\times\frac{2}{6})=\frac{14}{54}$ $p(L)=p(A\cap L)+p(\overline{A}\cap L)=\frac{5}{54}+\frac{14}{54}=\frac{19}{54}$	1.25
3	$p(A/L)=P\frac{(A\cap L)}{p(L)}=\frac{5}{19}$	0.5
4	$1-p(\text{none of the two dice shows 6})=1-\left[\left(\frac{1}{3}\times\frac{5}{6}\times\frac{5}{6}\right)+\left(\frac{2}{3}\times\frac{5}{6}\times\frac{4}{6}\right)\right]=\frac{43}{108}$. Or $p(L)+p(\text{two dice show the number 6})=\frac{19}{54}+\frac{2}{3}\times\left(\frac{1}{6}\times\frac{2}{6}\right)=\frac{43}{108}$	0.75

IV	Solution	M
1	$\lim\limits_{x\to-\infty} f(x) = -\infty-\infty=-\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty+0=+\infty$; $f(-1,5)$ □ -8.22 ,	0.75
2a	$f(x)-x = xe^{-x}$ $x>0;(C)$ is above (d) $x=0;(C)$ cuts (d) $x<0;(C)$ is below (d)	0.5
2b	$\lim\limits_{x\to+\infty}[f(x)-x] = \lim\limits_{x\to+\infty} xe^{-x} = 0$ then the line (d): y=x is an asymptote.	0.5
3	$f'(x)=1;\ 1+e^{-x}-xe^{-x}=1; x=1; A(1;1+e^{-1})$ $(T): y = x+e^{-1}$	1
4a	$f''(x)=0$ for $x=2$ and f'' changes sign. Then $W(2;2+2e^{-2})$ is a point of inflection to (C).	0.5
4b	f' admits an absolute minimum equals to $1-e^{-2}$ □ $0,8>0$ then f is strictly increasing. x: $-\infty$ … $+\infty$ $f'(x)$: + $f(x)$: $-\infty$ ↗ $+\infty$	0.5
5		1
6a	f is continuous and strictly increasing over □ ,then it admits an inverse functiuon h $D_h =]-\infty;+\infty[$	0.5
6b	(C') is the symmetric of (C) with respect to (d).	1
7a	$M(x;x+xe^{-x}); N(x+xe^{-x};x); MN=\sqrt{2}xe^{-x}$	1
7b	$MN=\sqrt{2}xe^{-x}=g(x)$ $g'(x)=0; \sqrt{2}e^{-x}(1-x)=0;$ for $x=1; MN=\sqrt{2}e^{-1}$ is the maximum value	0.75