Electromagnetic Induction

When the magnet moves w.r.t coil the magnetic field $\vec{B}$ created by the magnet at all points of the coil is varied. This variation of $\vec{B}$ sets up an electric current called an **induced current** in the coil galvanometer circuit.
The existence of such electric currents, in a **no** generator circuit, is associated with variation of physical quantity called **magnetic flux.**

$\vec{B}$
- $B_o = 4\pi \times 10^{-7} \times \frac{NI}{R}$, $N \rightarrow$ # turns
  $I \rightarrow$ intensity current
  $R \rightarrow$ radius of coil
- line of action : along axes of coil
  ⊥ to plane of coil at its center
- Direction : RHR

Unit of B is tesla (T)

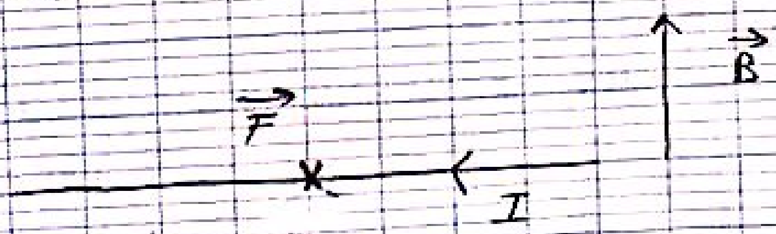
Laplace's force

$\vec{F} = \vec{I\ell} \wedge \vec{B}$

$\Rightarrow$ if $\vec{I}$ and $\vec{B}$ $\uparrow\uparrow$ then $\vec{F} = 0$

- We draw $\vec{F}$ at mdpt of wire

$$\vec{F} \begin{cases} F = I\,\ell\,B \sin(\vec{I},\vec{B}) \\ \perp \text{ to plane formed by } \vec{I} \text{ and } \vec{B} \\ \text{RHR} \end{cases}$$



Magnetic flux

$$\emptyset = \vec{B} \cdot \vec{n} \cdot S'$$

$\vec{B}$: magnetic induction vector of magnetic field

$\vec{n}$: normal vector ⊕

S' : total area of used coil

- Induction ⟹ influence without contact.

$$\emptyset = N\, B\, S \cos(\vec{B}, \vec{n})$$

$\emptyset$ = unit = weber (wb)

N = number of turns (m^2)

B = magnitude of $\vec{B}$ (T)

S = area of each coil

$$S' = N \times S$$

ϕ could be +ve −ve 0

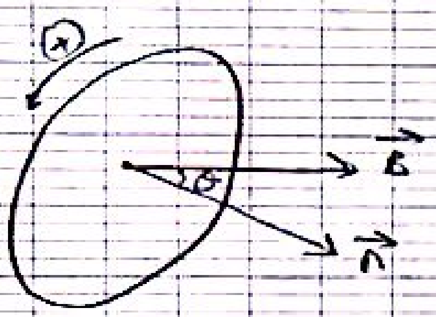
Depending on $\theta = (\vec{n}, \vec{B})$

① $0 \leqslant \theta \leqslant \frac{\pi}{2}$ $\cos\theta > 0$ $\phi > 0$

② $\theta = \frac{\pi}{2}$ $\cos\theta = 0$ $\phi = 0$

③ $\frac{\pi}{2} \leqslant \theta \leqslant \pi$ $\cos\theta < 0$ $\phi < 0$

①



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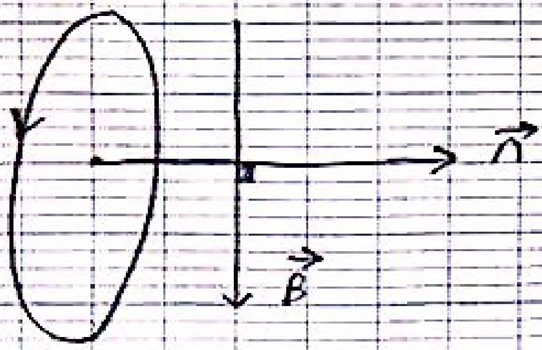

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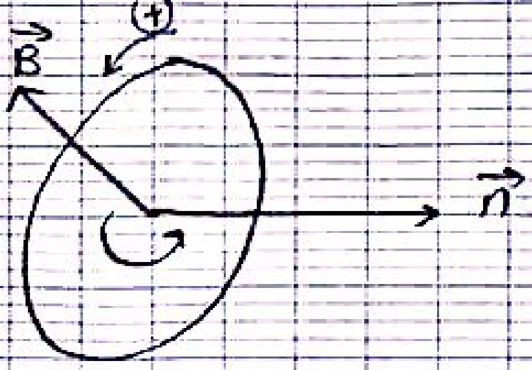


Conclusion :

The phenomenon observed where an electric current traverses a closed circuit crossed by a varying magnetic flux, is called electromagnetic induction. The circuit traversed by the induced current is called induced circuit. The source of the magnetic field, which is at the origin of the phenomenon of induction, is called inducing source.

Note

The establishment of an induced requires the existence of an induced electromotive force "e". A variation of magnetic flux through a circuit gives rise to an induced e.m.f "e" whether circuit closed or opened.

- If circuit closed ⇒ current
- If circuit opened ⇒ no current

Note

The variation of magnetic flux produces an induced voltage whether circuit opened or closed. But when circuit closed it produces induced current such that the current has electromagnetic effect opposes reason producing it.

Farady's Law

$$e = -\frac{d\phi}{dt}$$, $i r \rightarrow$ resistance of circuit
amp

induced voltage, which is the effect of variation of flux when circuit closed and current pass through.

Lenz's Law

Direction of induced current is such that its electromagnetic effect always oppose the cause producing it.

Note

Resistor $\Rightarrow$ $u = iR$

Generator $\Rightarrow$ $u = E - ir$, E = electromotive force

or if direction opp of current's Dir. then $u = ir - e$

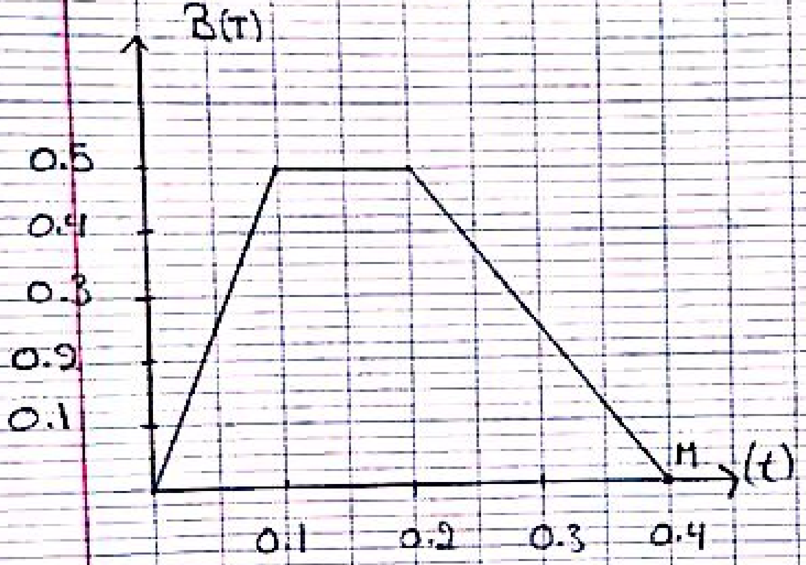
Coil $\Rightarrow$ $u = ir - e$

Exercise

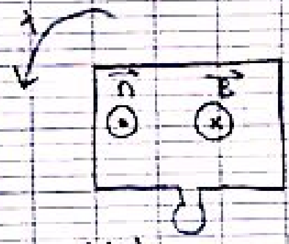
We place a squared coil of $a = 0.05\,m$ in a magnetic field with B as a function of time.

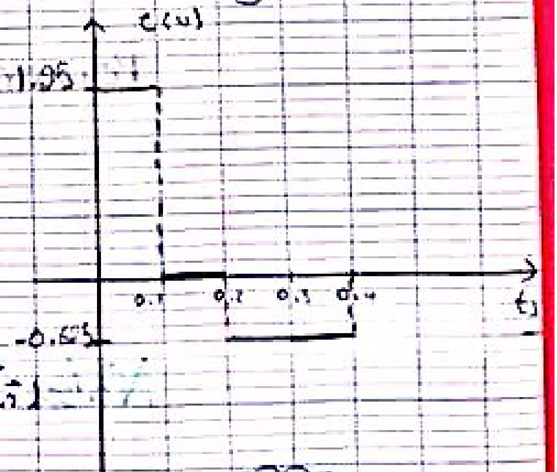
- Find e induced voltage / electromotive force.

We have :



① Expression of B wrt time
② Substitute B in eq. of $\varnothing$
③ Derive $\varnothing$ to get e
④ Deduce sign of i (from e)





- at $0 \leq t \leq 0.1$

$B = k \times t$ (passing origin)

$k = \text{slope} = \tan\alpha = \frac{0.5}{0.1} = 5$

$B_1 = 5t$

- at $0.1 \leq t \leq 0.2$

$B_2 = 0.5$ y cst // x axis

- at $0.2 \leq t \leq 0.4$

$B_3 = k't + b$ not thru origin $(y = ax + b)$

$k' = \frac{-\tan\beta'}{0.2} = -2.5$

$B_3 = -2.5t + b$

at M

$0 = -2.5(0.4) + b$

$b = 1$

$B_3 = -2.5t + 1$

- $\varnothing = BS\cos(\vec{B},\vec{n})$

$= -B.a^2$

$\varnothing_1 = -1.25t$, $e_1 = 1.25\,V$ ($i > 0$)

$\varnothing_2 = 0.125\,w$, $e_2 = 0$ ($i = 0$)

$\varnothing_3 = 0.625t - 0.25$, $e_3 = -0.625\,V$ ($i < 0$)

Notes from Exercises

$e = \phi'$ => stays same since derivative of lin. function cst
$\phi = kt + \phi_0$ linear function

if $I_1 \downarrow$ $B_1 \downarrow$ flux varies
then to increase B $\vec{B}_2$ flows in
same direction of $\vec{B}_1$ (Lenz's Law) ... RHR

Since ϕ cst -> no induced voltage and no induced current

Since oscilloscope has a very high resistance
=> there is a very small current flowing
in circuit which is negligible.

Power

$\underbrace{ei}_{\text{total electric power}} = \underbrace{ui}_{\text{useful or mechanical}} + \underbrace{i^2 R}_{\text{heat power by joules effect or dissipated}}$ (Ohm's law generator) $\times i$

Notes from Exercises

1 turn = 2π
turns / min $\xrightarrow{\div 60}$ turns / s
$(\vec{B}, \vec{n}) = \theta = \theta' t + \theta_0$

Electromagnetic Self Induction

$\phi = Li$

$L = 4\times10^{-7}\dfrac{N^2S}{l}$ (H) : Henry

The self induced electromotive force

$\Rightarrow e = -\dfrac{d\phi_p}{dt} = -L\dfrac{di}{dt}$

$U_{coil} = ri - e$
$= ri + L\dfrac{di}{dt}$

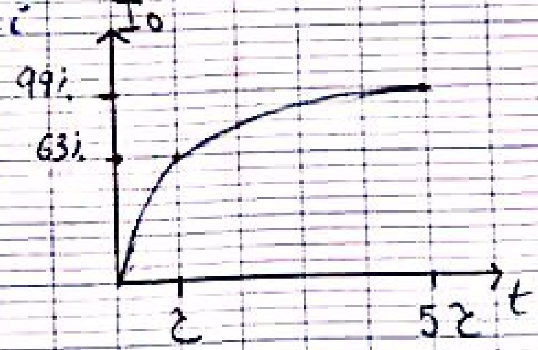
During growth phase

when I ↗ from zero to max ; flux varies due to magnetic field with increasing $\vec{B}$. It's called self flux But $\phi = B.NS$

$\cos(\vec{B},\vec{n}) = 0$
since B is also in $\vec{B}$ along axis of coil

$B = 4\times10^{-7}\dfrac{Ni}{l}$

$\Rightarrow \phi = 4\times10^{-7}\dfrac{N^2 i}{l} \Rightarrow \phi = Li$



at $t = 0$, $i = 0$

Diff equation

$E = Ri + L\dfrac{di}{dt}$

Solution:
$i = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$

$\tau = \frac{L}{R}$

and $I = \frac{E}{R}$

- $y = Ae^{x} \Rightarrow y' = A\, x' e$ • $\left(-\frac{t}{\tau}\right)' = -\frac{1}{\tau}$

- To verify a solution we substitute the given solution in the Diff eq.

ex: $i = \frac{E}{R} - \frac{E}{R} e^{-\frac{t}{\tau}}$

$E = iR + L\frac{di}{dt}$

. $i' = 0 - \frac{E}{R}\left(-\frac{1}{\tau}\right) e^{-\frac{t}{\tau}}$

. $i' = \frac{E}{R\tau} e^{-\frac{t}{\tau}}$

$E = \left(\frac{E}{R} - \frac{Ee^{-\frac{t}{\tau}}}{R}\right)R + L\left(\frac{E}{R\tau} e^{-\frac{t}{\tau}}\right)$

$E = E - Ee^{-\frac{t}{\tau}} + \frac{\cancel{L}}{\cancel{R}} \frac{E}{\cancel{\tau}} e^{-\frac{t}{\tau}}$

$E = E - Ee^{-\frac{t}{\tau}} + Ee^{-\frac{t}{\tau}}$

$E = E$ verified.

Scanned by CamScanner

if the circuit is totally resistive then $i = \frac{e}{R_{total}}$

i and e always of same sign

if circuit opened no current but still be an electromotive force

Right Hand Rules (RHR)

For Direction of $\vec{B}$

① Thumb toward current . Fingers toward pt . curl 90°
② Curl your fingers toward current (solenoid) . Your thumb is $\vec{B}$

For Direction of $\vec{F}$

Hold the wire . Thumb toward I. Place finger toward B . The third Finger is $\vec{F}$. Knowing that I and B are gun-shaped and $\vec{B}$ and $\vec{F}$ are $\perp$

Note

Area of 1 disk $\Rightarrow \frac{\pi d^2}{4}$

Exercise

Solution $E = iR + L\frac{di}{dt}$

$i = A(1-e^{kt})$

Determine A and K

$i = A - Ae^{kt}$

$\frac{di}{dt} = -Ake^{kt}$

$E = R(A - Ae^{kt}) - LAKe^{kt}$

$E = RA - RAe^{kt} - LAke^{kt}$

$E + 0e^{kt} = RA - Ae^{kt}(R + KL)$

By identification

$E = RA \Rightarrow A = \frac{E}{R}$

$A(R + KL) = 0$

$k = -\frac{R}{L}$

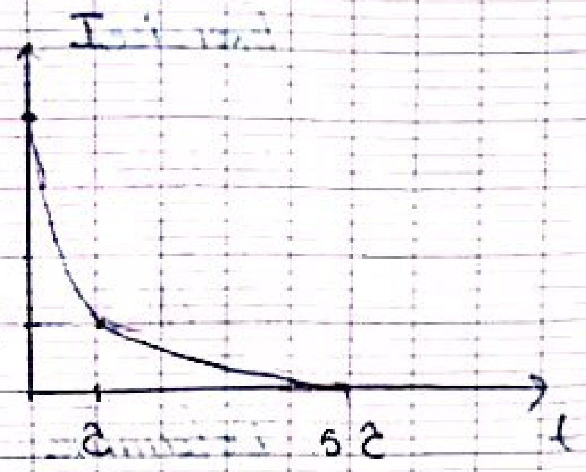
But $\tau = \frac{L}{R}$

$\frac{L}{R} = -\frac{1}{k} = \tau$

$k = -\frac{1}{\tau}$

$i = \frac{E}{R}(1 - e^{-\frac{t}{\tau}})$

During Decay Phase



Diff Equation

$$0 = iR + L\frac{di}{dt}$$

Solution

$$i = I e^{-\frac{t}{\tau}}$$

at $t = \tau$ $\Rightarrow i = 37\% I$

at $t = 5\tau$ $i = 0$

Notes

$$E_{magnetic} = \frac{1}{2} L i^2$$

stored or total

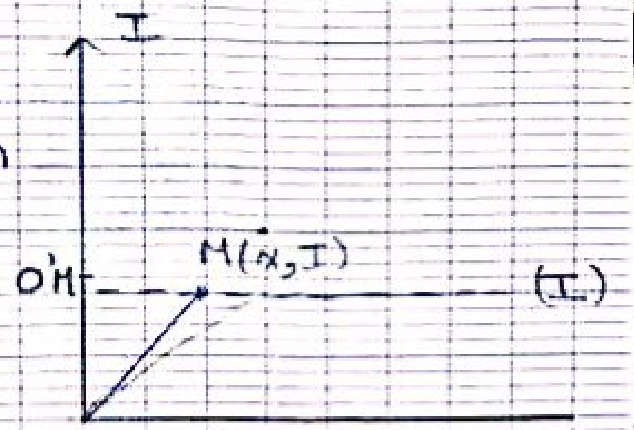
$$Ei = ri^2 + \frac{d\left(\frac{1}{2} L i^2\right)}{dt}$$

electric energy — heat energy — electromagnetic energy at certain instant

During Decay phase when self induced emf same direction of current $\Rightarrow$ coil plays role of generator sens

During Growth phase when self induced emf opp direction (sign) of current $\Rightarrow$ coil receiver or generator opposition

Use graph to Determine: τ (time ct)



Tangent Passing Through Origin

$y = At$

slope = derivative of function

Given: $i = I(1-e^{-\frac{t}{\tau}})$

$\frac{di}{dt} = \frac{I}{\tau} e^{-\frac{t}{\tau}}$

at $t=0$ $\frac{di}{dt} = \frac{I}{\tau}$

Substitute M (x, I) in $y = \frac{I}{\tau} t$

$I = \frac{I}{\tau} \cancel{\tau} x$

$\frac{x}{\tau} = 1 \Rightarrow x = \tau$

$x = \overline{O'M} = \tau$

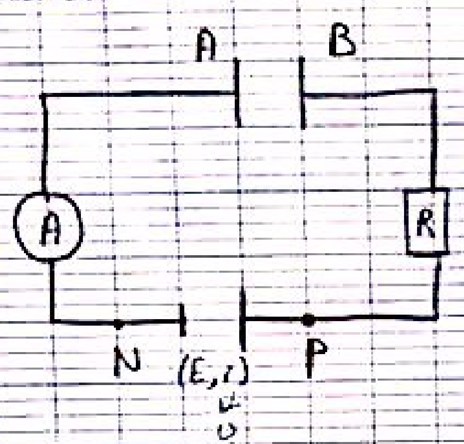
Notes

$\cos' = -\sin$

$\sin = \cos$

Capacitor

Its a system that stores electric energy. Its made up of 2 conducting plates seperated by insulator



P is +ve (needs e^-) and is connected to B. B can't always be neutral so it loses part of its e^- and gives them to plate P in such a way to reach equilibrium

$\Rightarrow U_{PB} = 0$

When plate B becomes positive A becomes -vely charged due to accepting e^- from plate P

Notes

Charge of capacitor

- $q = q_B = -q_A$
- $q = C . U_{AB}$
- if I connect capacitor to device it acts as if generator

Quantity of Stored Energy

$$W = \frac{1}{2} C U^2$$
$$= \frac{1}{2} \frac{q^2}{C}$$
$$= \frac{1}{2} q . U$$

Notes

$q = ne \rightarrow 1.6 \times 10^{-19}$

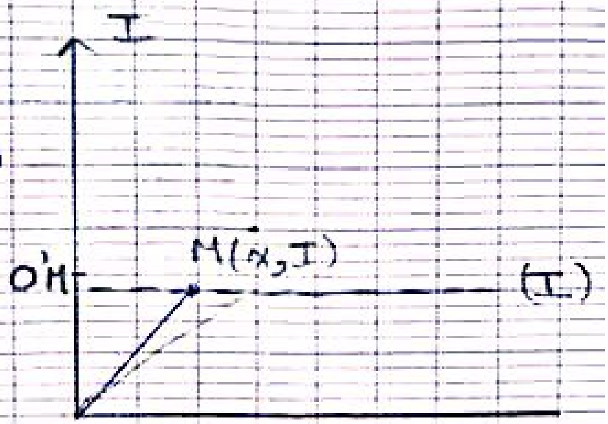
$q = It$ if current cte

- $i = \frac{dq}{dt}$

$i = C \frac{du}{dt}$

Use graph to Determine τ (time ct)

Tangent Passing Through Origin



$y = At$

slope = derivative of function

Given: $i = I(1 - e^{-\frac{t}{\tau}})$

$$\frac{di}{dt} = \frac{I}{\tau} e^{-\frac{t}{\tau}}$$

at $t = 0$ $\frac{di}{dt} = \frac{I}{\tau}$

Substitute $M(x, I)$ in $y = \frac{I}{\tau} t$

$$I = \frac{I}{\tau} x$$

$$\frac{x}{\tau} = 1 \Rightarrow x = \tau$$

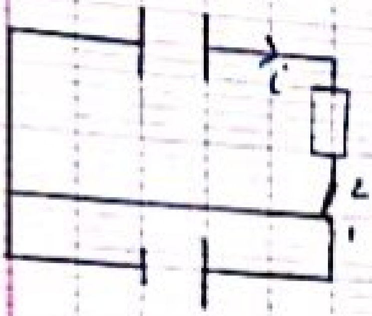
$$x = \overline{O'M} = \tau$$

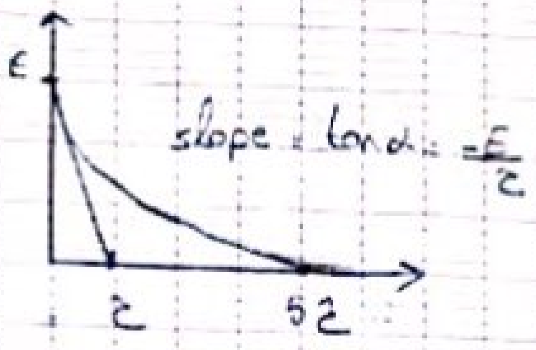
Notes

$\cos' = -\sin$

$\sin' = \cos$

Discharging





current opp Direction as before (charging)

Diff eq

$iR = u_C$, but $i = -C\frac{du}{dt}$

$u = -RC\frac{du}{dt}$

$u + RC\frac{du}{dt} = 0$

Solution

$u = Ee^{-\frac{t}{\tau}}$

left

at $t = \tau \Rightarrow u = 37\%E$

$q = 37\% Q$

at $t = 5\tau \Rightarrow u = 0.007 \approx 0$

$q = 0$

Note

$i = -I_0 e^{-\frac{t}{\tau}}$

Note

at the end of charging or Discharging the current in capacitor is zero ($i = 0$)

Alternating Sinosoidal Current

- $i = I_m \sin(\omega t + \varphi)$

i → inst intensity; I_m → max intensity; ω → angular freq; φ → phase at $t=0$

- $u = U_m \sin(\omega t + \varphi)$

u: inst voltage; U_m → max voltage; ω → equal angular freq; φ → phase at $t=0$ (not necessary equal)

effective current

$$I = \frac{I_m}{\sqrt{2}}$$

effective voltage

$$U = \frac{U_m}{\sqrt{2}}$$

Notes : Rules

Resistor : $u = iR$

Coil (r, L) : $u = ir + L\frac{di}{dt}$

Capacitor : $i = \frac{dq}{dt} = C\frac{du}{dt}$

Pure Resistor R

$i = I_m \sin \omega t$ reference

$\Rightarrow u = U_m \sin(\omega t + \varphi)$

- $U_m = RI_m$, $U = RI$

$\varphi = 0 \Rightarrow$ i in phase with u

Coil $(L, 0)$

$i = I_m \sin(\omega t)$

$\Rightarrow u = U_m \sin(\omega t + \varphi)$

$U_m = I_m L\omega$, $U = IL\omega$

$\varphi = \frac{\pi}{2} \Rightarrow$ u_L leads i by $\frac{\pi}{2}$

Capacitor C

$i = I_m \sin \omega t$

$\Rightarrow u = U_m \sin(\omega t + \varphi)$

$U_m = \frac{I_m}{C\omega}$, $U = \frac{I}{C\omega}$

$\varphi = -\frac{\pi}{2} \Rightarrow$ u_C lags i by $\frac{\pi}{2}$

Rule of Calculation

$$i = c\frac{du}{dt}$$

$$du = \frac{i}{c}\,dt$$

$$\int du = \int \frac{i}{c}\,dt$$

$$u = \int \frac{I_m \sin(\omega t)}{c}\,dt$$

$$u = \frac{I_m}{c\omega}(-\cos \omega t) + k \qquad \text{at } t=0 \quad k=0$$

$$u = -\frac{I_m}{c\omega}\cos \omega t$$

$$u = -\frac{I_m}{c\omega}\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$u = \frac{I_m}{c\omega}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$U_m \sin(\omega t + \varphi) = \frac{I_m}{c\omega}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$U_m = \frac{I_m}{c\omega} \quad , \quad \varphi = -\frac{\pi}{2}$$

$\Rightarrow$ u lags i by $\frac{\pi}{2}$

Note from Exercise

$u_R = 2.4 \sin(400\pi t + 0.4\pi)$

u capacitor lags i by $\frac{\pi}{2}$

R is image of i

$\Rightarrow u_C = \frac{I_m}{C\omega} \sin(\omega t + 0.4\pi - \frac{\pi}{2})$

Powers (Average)

$\boxed{P_{resistor} = P_{generator} = RI^2}$

in AC voltage : $\boxed{P_{av} = UI \cos\varphi}$

Capacitor $|\varphi| = \frac{\pi}{2}$

$\boxed{P = 0}$

Coil of $r = 0$ $|\varphi| = \frac{\pi}{2}$

$\boxed{P = 0}$

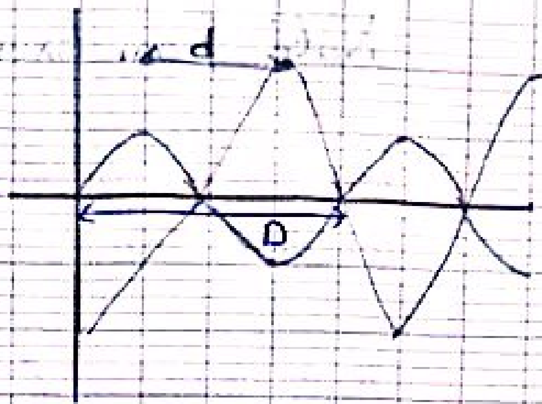
Note

in (R-L-C)

proper freq $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Note

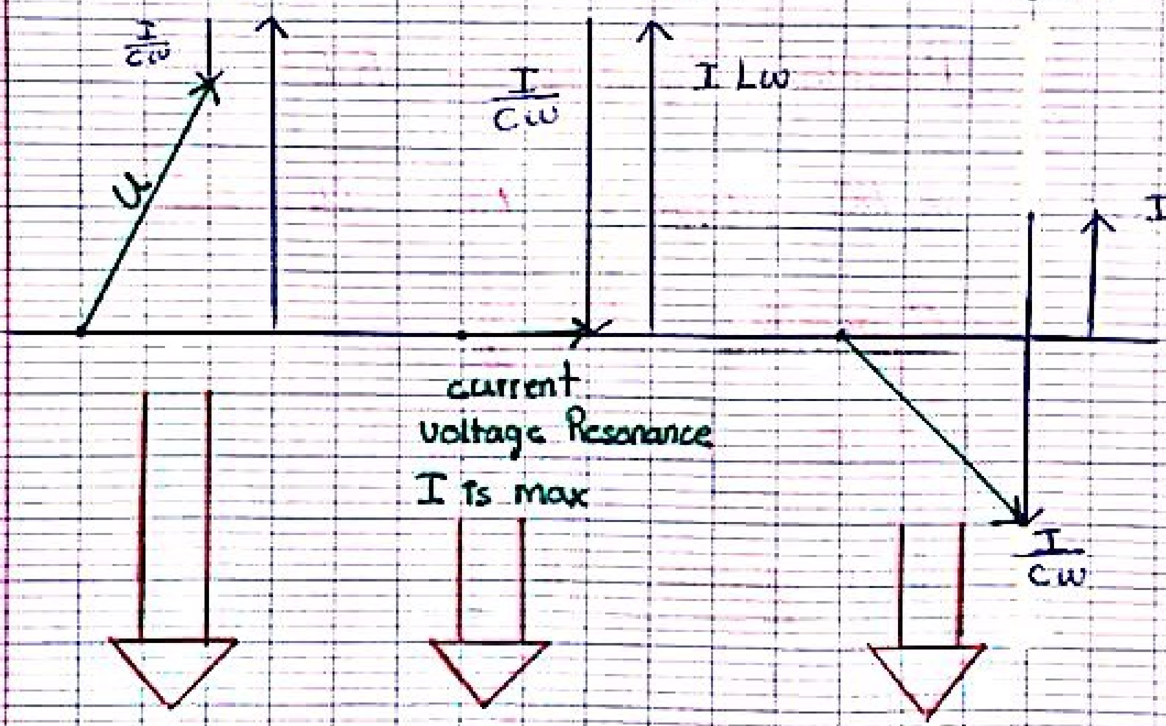
$$\varphi = \frac{2\pi d}{D}$$



$$\begin{cases} D = 2\pi \\ d = \varphi \end{cases} \Rightarrow \varphi = \frac{2\pi d}{D}$$

Note

$L\omega > \frac{1}{C\omega}$ $\qquad$ $L\omega = \frac{1}{C\omega}$ $\qquad$ $\frac{1}{C\omega} > L\omega$



u leads i $\qquad$ Resonance => in phase $\qquad$ u lags i

Note

from Degree to rd

- $180 \longrightarrow 3.14$ rd
  $33.7 \longrightarrow x$

- $\omega = 2\pi f$