Functions

$\frac{k}{\infty} = \mathbf{0}$ $\frac{k}{0} = \infty$ $\frac{\infty}{0} = \infty$

$\frac{\infty}{\infty}$ or $\frac{0}{0}$→ Apply L'Hopital's Rule

Which means $\lim \frac{u}{v} = \lim \frac{u'}{v'}$

$\infty - \infty$ indetermined form

take the stronger common factor.

Lnx<<< x <<< e^x

$0.\infty$ indetermined form

$\frac{\infty}{\frac{1}{0}} = \frac{\infty}{\infty}$ then apply L'Hopital's Rule

$\infty + \infty = \infty$ $-\infty - \infty = -\infty$

$k.\infty = \infty$ $\frac{0}{\infty} = 0$. $\frac{1}{\infty} = 0$

- $\lim_{x \to k} f(x) = \pm\infty$ → x = k is vertical asymptote.
- $\lim_{x \to \infty} f(x) = k$ → y = k is horizontal asymptote.
- To prove (d): y = ax + b is oblique asymptote

$\lim_{x \to \infty} [\ f(x) - y\ ] = 0$

- **To study the relative position of (C) and (d):**

  ⇒ Check the sign of f(x) – y

  - f(x) – y > 0 → (C) is above (d).
  - f(x) – y < 0 → (C) is below (d).
  - f(x) – y = 0 → (C) intersects (d).

03-227888

- **equation of the tangent to (C) at x = a:**

  $y = f(a) + (x-a).\, f'(a)$

- To find the point of intersection between 2 curves, put $f(x) = g(x)$ then find x.
- Point of inflection $\rightarrow f''(x) = 0$ and f'' changes sign.
- I (a,b) is a center of symmetry then, $f(2a-x) + f(x) = 2b$
- (d): $x = a$ is an axis of symmetry:

  $f(2a - x) = f(x)$.

- $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

  $(u.v)' = u'v + v'u$

  $(u^n)' = n.u^{n-1}.u'$

- **To prove f admits an inverse function:**

  f is defined, continuous and strictly monotonic (↑ or ↓), then f admits an inverse function whose graph is symmetric w.r.t the line $y = x$.

  - $D_{f^{-1}} = R_f$
  - f and f' have the same variation.
  - To find the point of intersection between the function and its inverse, put $f(x) = x$ and solve to find x.
  - To prove g(x) in the inverse of f (x), then $f(g(x)) = x$.

- Tangent parallel to the x-axis $\rightarrow f'(x) = 0$.
- Slope of the tangent = f' (point of tangency).
  - $(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$

    * To prove A(a,b) belongs of $f^{-1}(x)$, prove $f(b) = a$

- Sign of f (x):

  * If the graph of f (x) is above the x-axis, then $f(x) > 0$.
  * If the graph of f (x) is below the x-axis, then $f(x) < 0$.
  * If the graph of f (x) intersects the x-axis, then $f(x) = 0$.

03-227888

- **To prove f(x) = 0 admits a unique root $\propto \in$] a, b [:**

  1- f is defined, continuous and strictly monotonic.

  2- f(a) . f(b) <0

- odd function: f (-x) = - f (x) $\Rightarrow$ symmetric w.r.t. O (0,0)
- even function: f (-x) = f (x) $\Rightarrow$ symmetric w.r.t. y- axis.
- If the graph of f admits maximum of minimum at x = a , then f ′(a) = 0.

- $\lim\limits_{x \to 0+} lnx = -\infty$ $\qquad$ $\lim\limits_{x \to +\infty} lnx = +\infty$

- ln 1= 0 $\qquad$ ln e= 1

  (ln x)′ = $\frac{1}{x}$ $\qquad$ (ln u)′ = $\frac{U'}{u}$

  ln (a.b) = ln a + ln b

  ln ($\frac{a}{b}$) = lna – lnb

  Ln (a^n) = nln a

  Ln e^x = x $\qquad$ $e^{\ln x}$ =x

- e^0 = 1 $\qquad$ $e^{+\infty} = +\infty$ $\qquad$ $e^{-\infty} = 0$

  $e^x.e^y = e^{x+y}$ $\qquad$ e^x> 0 for any real number x.

  $\frac{e^x}{e^y} = e^{x-y}$

  $(e^x)' = e^x$

  $(e^u)' = u' . e^u$

- lnx = k $\Leftrightarrow$ x = e^k

  e^x = k $\Leftrightarrow$ x = lnk