Complex Numbers

- Algebraic form of a complex number:
  Z = x + iy
- Z is the affix of point M(x,y).
- Trigonometric form: z = r ($\cos\theta$+ i$\sin\theta$).
  Where r = $\sqrt{x^2 + y^2}$.

To find θ: put $\tan\theta = \left|\frac{y}{x}\right|$.

Then on calculator $\tan^{-1}$(ans) = α

If (x,y) = (+, +) →$\theta = \alpha$

(x,y) = (+, -) →$\theta = -\alpha$

(x,y) = (-, -) →$\theta = \pi + \alpha$

(x,y) = (-, +) →$\theta = \pi - \alpha$

Arg (z) = $\theta + 2k\pi$

- Exponential form : z = $re^{i\theta}$
- $|Z_B - Z_A| = AB$
- If M belongs to a circle of center A and radius r, then $|Z_M - Z_A| = r$.
- If MA = MB, then M moves on the perpendicular bisector of [AB].
- If MA = k, then M moves on a circle of center A and radius k.
- Z.$\overline{Z}$ = $x^2 + y^2$ = $|Z|^2$
  where $\overline{Z}$ is the conjugate of Z
  $\overline{Z}$ = x –iy
- Arg ($Z_1.Z_2$) = Arg(Z_1) + Arg(Z_2)
  Arg ($\frac{Z_1}{Z_2}$) = Arg(Z_1) -Arg(Z_2)
  Arg (Z^n) = nArg(Z)
  Arg ($\overline{Z}$) = -Arg(Z)
- $|Z| = |\overline{Z}|$

- $Z = X + i\,\gamma$ $\quad$ X = real part $\quad$ γ = imaginary part
- If Z is real $\rightarrow$ imaginary part = o
- If Z is pure imaginary $\Rightarrow$ real part = 0

  And imaginary part $\neq 0$

- To prove A,B,C are collinear

  $\frac{Z_B - Z_A}{Z_C - Z_A}$ is real.
- To prove ΔABC is right at A:

  $\frac{Z_B - Z_A}{Z_C - Z_A}$ is pure imaginary

- To prove (AB) ⇈(CD): $\frac{Z_B - Z_A}{Z_D - Z_C}$ is real.
- Arg $(Z_M) = (\vec{U}, \overrightarrow{OM}) + 2k\pi$
- Arg $(Z_B - Z_A) = (\vec{U}, \overrightarrow{AB}) + 2k\pi$
- Arg $(\frac{Z_B - Z_A}{Z_C - Z_A})$ = Arg $(Z_B - Z_A)$ - Arg $(Z_C - Z_A)$

  $= (\vec{U}, \overrightarrow{AB}) - (\vec{U}, \overrightarrow{AC})$

  $= (\overrightarrow{AC}, \overrightarrow{AB}) + 2k\pi$
- $Z = -r\,[\cos\alpha + i\sin\alpha] = r[\cos(\pi + \alpha) + i\sin(\pi + \alpha)]$

  $Z = r\,[\cos\theta - i\sin\theta] = r[\cos(-\theta) + i\sin(-\theta)]$

  $Z = r\,[\sin\theta + i\cos\theta] = r[\cos(\frac{\pi}{2} - \theta) + i\sin(\frac{\pi}{2} - \theta)]$

  $\overline{Z_1 + Z_2} = \overline{Z_1} + \overline{Z_2}$ $\quad$ $\bar{i} = -i$

- $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$
- $\cos\theta = \frac{e^{i\theta} - e^{-i\theta}}{2}$

  $\sin\theta = \frac{e^{i\theta} + e^{-i\theta}}{2i}$
- if Z is real, then arg(z) = $k\,\pi$

  if Z is pure imaginary, then arg(z) = $\frac{\pi}{2} + k\,\pi$
- $e^{i\theta} . e^{+i\alpha} = e^{i(\theta + \alpha)}$

$\frac{e^{i\theta}}{e^{i\alpha}} = e^{i(\theta - \alpha)}$ $(e^{i\theta})^n = e^{in\theta}$

$\overline{Z} = re^{-i\theta}$

- If Z is real $\Leftrightarrow$ Z = $\overline{Z}$

  If Z is pure imaginary $\Leftrightarrow \overline{Z}$ = -Z