Test(1)

I) In this question, all steps of calculations must be shown.
Given the numbers:

$$A = \frac{5}{9}\times\left(\frac{1}{2}+\frac{3}{10}\right)-\frac{2}{3}\left(1-\frac{5}{2}\right)\ ;\ B=\frac{(25\times6)^2}{10^5\times9^2}\ ;\ C=\left(2\sqrt{5}+2\right)\left(\sqrt{5}-2\right);\ D=\left(\sqrt{5}-1\right)^2;\ E=\frac{8}{3+\sqrt{5}}$$

1) Calculate A, write the answer in the form of an irreducible fraction.
2) Calculate B, write the answer in the form of $2^x\times3^y\times5^z$ where x, y and z are three consecutive integers.
3) Show that C=D=E.

II) Given the expression $S = 2x^2 - 3x - 9$
1) Copy and complete this table:

Value of x	- 1.5	0	3	3.5
Value of S	0			

2) Deduce the solutions of the equation S=0.
3) In this part, x represents a length expressed in cm.
ABCD is a rectangle of dimensions 2x + 3 and x – 3.
Give the value of x for which the area of ABCD is equal to 5 cm^2.

III) 1) Solve the inequality: $x+15\geq\frac{2}{3}(x+27)$.

2) A research office employed 27 computer scientists and 15 mathematicians. The office plans to increase the same number x of computer scientists and mathematicians.
How many specialists of each type should the office employ so that the number of mathematicians is at least two thirds the number of computer scientists?

IV) Last weekend, two customers entered the same store.
The first customer bought six balls and a garland, he paid 18400 LL.
A second customer has a loyalty card that entitling him to get a 20% discount on all items.
He bought five balls and five garlands. He paid, using his loyalty card at checkout, 25600 LL.
The aim of this problem is to find the price of a ball and a garland.

1) a) By considering the purchase of the first client, explain what do 6x and y represent when we write the equation 6x + y = 18400. Specify the unit of x and y.
b) Deduce the price of 60 balls and 10 garlands.

2) a) Explain why when we have a 20% discount, we multiply the price by 0.8.
b) By considering the purchase of the second customer, what equation can we write?
Show that it can be written as: x + y = 6400.

3) Answer the problem by making the necessary calculations.

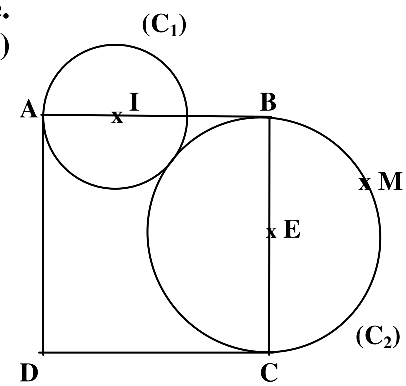
V) To do a Math test, the 35 students of a class are divided into two groups A and B.
1) Here is the list of marks obtained by group A students:
15 ; 10 ; 8 ; 8 ; 12 ; 10 ; 10 ; 9 ; 7 ; 12 ; 15 ; 10 ; 7 ; 8 ; 12 .

a) Construct the table of frequencies and increasing cumulative frequencies for the list of marks.
b) What is the percentage of students who took a mark greater than or equal to 10?
c) Determine the mean mark of group A students.

2) Group B students received an average mark of 11.7.
What is the average mark obtained by the class?

VI) On an orthonormal system of axes x'Ox, y'Oy, where the unit of length is the cm, consider the points $A\left(2;\frac{5}{2}\right)$, $B\left(6;\frac{9}{2}\right)$ and $C\left(3;\frac{3}{2}\right)$.

1) Plot the points A, B and C.
2) a) Calculate the lengths AC and BC.
b) Knowing that $AB=2\sqrt{5}$, prove that ABC is a right triangle at C.
3) Calculate the coordinates of E the midpoint of [AB].
4) D is the image of C by the translation of vector $\vec{u}(2;0)$.
a) Plot D then show that the coordinates of D are $\left(5;\frac{3}{2}\right)$.
b) Calculate DE then deduce that ABD is a right triangle in D.
5) a) Show that the equation of the line (DE) is $y=-2x+\frac{23}{2}$.
b) Calculate the slope of (AB).
c) Deduce that (AB) and (DE) are perpendicular.
6) F is a point defined by $\overrightarrow{EF}=\overrightarrow{DE}$.
Place F then prove that the quadrilateral ADBF is a square.
7) Show that the points A, B, C, D and F are on same circle (C) whose center and radius are to be determined.



VII) ABCD is a square of side 6 cm.
E is the midpoint of [BC].
I is a point on [AB] distinct from A and B.
(C_1) is the circle of center I and passing through A.
(C_2) is the circle of diameter [BC].
(C_1) and (C_2) are tangent externally.

Part I:
Denote by x the distance AI where x is a length expressed in cm.
1) Use Pythagoras theorem in triangle IBE to express IE^2 as function of x.
2) Show that x satisfies the equation $(x + 3)^2 = (6 - x)^2 + 9$.
3) Deduce the position of I on [AB].

Part II:
Suppose in what follows that x=2 cm.
1) Reproduce the figure.
2) What does (AD) represent for (C_1)? Justify the answer.
3) (AB) cuts (C_1) in another point G. Prove that (GE) and (IC) are parallel.
4) H is the center of ABCD. Prove that H is on (C_2).
5) a) Prove that the two triangles ABH and ACB are similar.

b) Deduce that AC= $\sqrt{2}\times$AB.

6) M is a variable point on (C_2) and N is the symmetric of B with respect to M.

a) Show that BCN is an isosceles triangle.

b) Deduce the locus of N as M moves on (C_2).

Correction of Test(1)

I) 1) $A=\frac{5}{9}\times\left(\frac{1}{2}+\frac{3}{10}\right)-\frac{2}{3}\left(1-\frac{5}{2}\right)=\frac{5}{9}\times\left(\frac{5+3}{10}\right)-\frac{2}{3}\left(\frac{2-5}{2}\right)=\frac{5}{9}\times\frac{8}{10}-\frac{2}{3}\times\frac{-3}{2}=\frac{4}{9}+\frac{1}{1}=\frac{4+9}{9}=\frac{13}{9}$

2) $B=\frac{(25\times6)^2}{10^5\times9^2}=\frac{\left(5^2\times2\times3\right)^2}{(2\times5)^5\times\left(3^2\right)^2}=\frac{5^4\times2^2\times3^2}{2^5\times5^5\times3^4}=2^{-3}\times3^{-2}\times5^{-1}$

3) $C=\left(2\sqrt{5}+2\right)\left(\sqrt{5}-2\right)=10-4\sqrt{5}+2\sqrt{5}-4=6-2\sqrt{5}$; $D=\left(\sqrt{5}-1\right)^2=\left(\sqrt{5}\right)^2-2\left(\sqrt{5}\right)(1)+(1)^2=6-2\sqrt{5}$

$E=\frac{8}{\left(3+\sqrt{5}\right)}\times\frac{\left(3-\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}=\frac{8\left(3-\sqrt{5}\right)}{(3)^2-\left(\sqrt{5}\right)^2}=\frac{8\left(3-\sqrt{5}\right)}{4}=6-2\sqrt{5}$ So C = D = E.

II) 1)

Value of x	- 1.5	0	3	3.5
Value of S	0	9	0	5

2) S=0 for x = - 1.5 or x = 3 (from the table).

3) Area= L×W=(2x + 3)(x – 3) = $2x^2 - 3x - 9$ = S & S=5 for x = 3.5 cm (from the table).

III) 1) $x+15\geq\frac{2}{3}(x+27)$ then $x+15\geq\frac{2}{3}x+18$ or $x-\frac{2}{3}x\geq18-15$ then $\frac{1}{3}x\geq3$ so $x\geq\frac{\frac{3}{1}}{\frac{1}{3}}\geq9$.

2) Let x be the number of specialists then $x+15\geq\frac{2}{3}(x+27)$ then $x\geq9$ so the office should employ at least 9 specialists of each type.

IV) 1) a) 6x represents the price of six balls in LL and y represents the price of a garland in LL.

b) 6x + y = 18400 so 10(6x + y)=10(18400) then 60x+10y=184000 LL is the price of 60 balls and 10 garlands.

2) a) 100% - 20%=80%=0.8 so we multiply the price by 0.8.

b) 0.8(5x) + 0.8(5y)=25600 then 4x + 4y=25600 or x + y = 6400.

3) $\begin{cases} 6x + y = 18400 \\ x + y = 6400 \end{cases}$; solving the system gives x=2400 LL is the price of a ball and y=4000 LL is the price of garland.

V) 1) a)

value	7	8	9	10	12	15	Tot
freq	2	3	1	4	3	2	15
ICF	2	5	6	10	13	15	

b) $\text{percentage} = \dfrac{\text{freq}}{\text{tot}} \times 100 = \dfrac{9}{15} \times 100$ =60%.

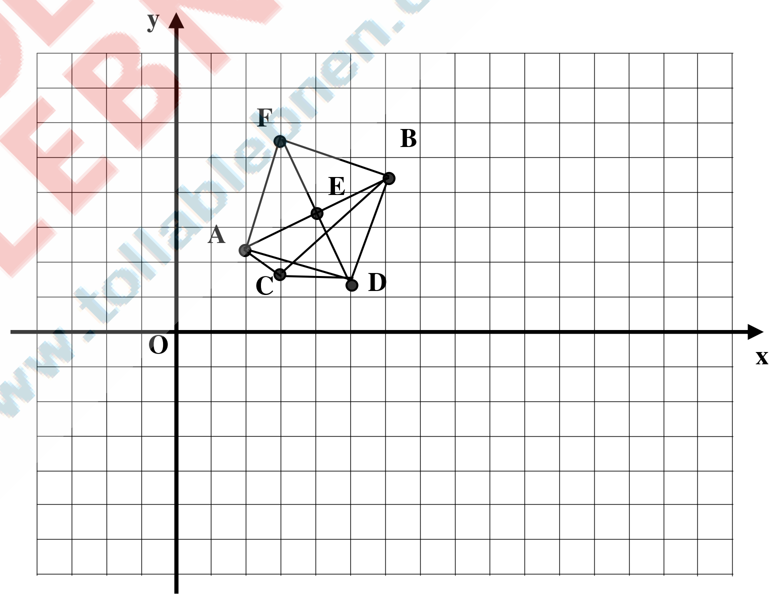
c) $\text{mean} = \dfrac{7 \times 2 + 8 \times 3 + \text{---}}{15}$ =10.2.

2)

	(A)	(B)	
mean	10.2	11.7	Tot
freq	15	20	35

$\text{average} = \dfrac{10.2 \times 15 + 11.7 \times 20}{35}$ =11.05

VI) 1) see the figure.



2) a) $AC = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2} = \sqrt{2}$ cm and $BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = 3\sqrt{2}$ cm.

b) Apply the converse of Pythagoras' theorem.

$CA^2 + CB^2$; AB^2 then 20=20 so $CA^2 + CB^2 = AB^2$ then ABC is a right at C.

3) $x_E = \dfrac{x_A + x_B}{2} = 4$ and $y_E = \dfrac{y_A + y_B}{2} = \dfrac{7}{2}$ so $E\left(4; \dfrac{7}{2}\right)$.

4) a) $\overrightarrow{CD} = \vec{u}$ then $x_{\overrightarrow{CD}} = x_{\vec{u}}$ or $x_D - x_C = 2$ then $x_D = 5$

and $y_{\overrightarrow{CD}} = y_{\vec{u}}$ or $y_D - y_C = 0$ then $y_D = \frac{3}{2}$.

b) $DE = \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2} = \sqrt{5}$ cm then $DE = \frac{1}{2}AB = \frac{1}{2}$hyp then ABD is right D.

5) a) Equation of (DE) is $y=a(x - x_E) + y_E$, $a = \frac{y_E - y_D}{x_E - x_D} = -2$ then $y=-2(x-4) + \frac{7}{2}$ or (DE): $y= -2x + \frac{23}{2}$.

b) $a_{(AB)} = \frac{y_B - y_A}{x_B - x_A} = \frac{1}{2}$.

c) $a_{(AB)} \times a_{(DE)} = \frac{1}{2} \times (-2) = -1$ then (AB) is perpendicular to (DE).

6) DE=EF (by translation), AE=EB (given) and DE=EB=EA (proved) so the diagonals [AB] and [FD] bisect each other at E, perpendicular and equal, then ADBF is a square.

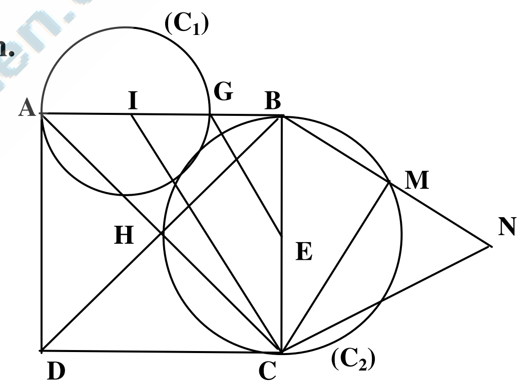
7) ABC, AFB and ABD are three right triangles of same hyp [AB], then the points A, B, C, D and F are on same circle (C) whose center is E the midpoint of [AB].

VII) Part I:

1) $IE^2 = IB^2 + BE^2 = (6 - x)^2 + (3)^2 = (6 - x)^2 + 9$.

2) IE= $R_1 + R_2 = x + 3$ and $IE^2 = (6 - x)^2 + 9$ so $(x + 3)^2 = (6 - x)^2 + 9$.

3) $(x + 3)^2 = (6 - x)^2 + 9$ then $x^2 + 6x + 9 = 36 - 12x + x^2 + 9$ or x= 2 then the position of I is on [AB] where AI=2 cm.



Part II:

1) See the figure.

2) (AD) is perpendicular to the diameter [AB] at A then (AD) is tangent to (C_1) at A.

3) GB= 6–4=2 then (GE) // (IC) (midpoint theorem).

4) $\widehat{BHC} = 90^o$ (diagonals in square) then HE=EB=EC=R_2 so H is on (C_2).

5) a) $\widehat{HAB} = \widehat{CAB}$ (same angle) and $\widehat{AHB} = \widehat{ABC} = 90^o$ so ABH and ACB are similar by AA.

b) Ratio of similarity: $\frac{AB}{AC} = \frac{AH}{AB} = \frac{BH}{BC}$ then $\frac{AB}{AC} \times \frac{AH}{AB}$ or $AC \times AH = AB^2$

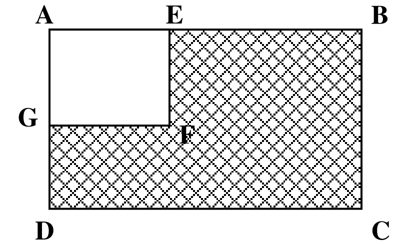
but $AH = \frac{1}{2}AC$ so $AC \times \frac{1}{2}AC = AB^2$ or $AC^2 = 2AB^2$ then $AC = \sqrt{2AB^2} = \sqrt{2} \times AB$.

6) a) $\widehat{BMC} = 90^o$ (inscribed in semicircle) then (CM) is the perpendicular bisector of [BN] then BCN is an isosceles triangle of vertex C.

b) C is a fixed point and CN=CB=6 cm then the locus of N is the circle of center C and radius 6 cm.

Test(2)

I) The parts 1) and 2) are independent.



1) ABCD is a rectangle and AEFG is a square where DC= $5\sqrt{21} - \sqrt{3}$ cm , BC= $7\sqrt{3}$ cm and AG= $3\sqrt{2}$ cm.

Calculate the exact area of the shaded figure.

2) Consider the number $A = \left(\frac{p^2+1}{2}\right)^2 - \left(\frac{p^2-1}{2}\right)^2$ where P is a prime number greater than 2.

a) Show that A is a square of a prime number.

b) Deduce the two consecutive integers a and b where $23^2 = a^2 - b^2$.

II) Consider the expressions $A(x) = (2x-5)^2 - 3(x-1)(2x-5)$ & $B(x) = (x+3)(x+2) - (2x+4)x + (x+2)$.

1) a) Factorize A(x).

b) Solve the equation A(x) = 0.

2) a) Develop & reduce B(x).

b) Solve the equation B(x) = 8.

c) Show that $B(x) = (x+2)(4-x)$.

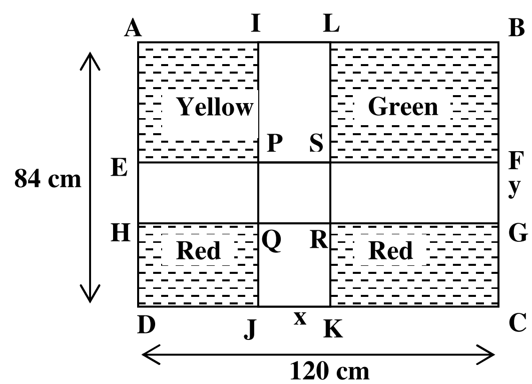
3) Consider the equation (E): $\frac{A(x)}{B(x)} = \frac{3}{2}$.

a) For what values of x the equation (E) is defined?

b) Show that (E) is equivalent to the equation $2(2x-5)=3(x-4)$.

c) Does (E) admit a solution? Justify the answer.

III) Consider the rectangular board ABCD of length 120 cm and width 84 cm. The board is divided into 4 painted rectangles that are separated by two crossed rectangles of respective widths x and y (expressed in cm) as shown in the figure. Denote by S the painted surface area.



Part A:

In this part, we know that:

- 35% of S is painted green
- 25% of S is painted yellow
- the area of the red rectangles is 2688 cm^2.

1) Calculate S.

2) What percentage of the total area does S represent? Give the answer to nearest tenth.

3) a) Verify that the area covered by the crossed rectangles, express in terms of x and y, is $84x + 120y - xy$.

b) Knowing that $x = \frac{1}{8}AB$, find x then deduce y.

Part B:

In this part, given that x= 12 cm and y is a strictly positive integer expressed in cm. Determine the possible values of y so that the area of the crossed rectangles remains less than 15% of the total surface area of the board.

IV) Consider the plane of an orthonormal system of axes x'Ox;y'Oy, where the unit of length is the cm, the straight line (Δ) of equation $y = 2x - 3$ and the points A(-4 ; 7) & B(8 ;1).

1) a) Draw the line (Δ).
   b) (Δ) cuts the ordinate axis at E. Calculate the coordinates of E.
2) Show that the equation of the line (AB) is $y = -\frac{1}{2}x + 5$.
3) S is the intersection point of (AB) & (Δ).
   a) Prove that (AB) & (Δ) are perpendicular.
   b) Prove that K(4 ;-1) is the center of the circumscribed circle,(C), about SEB.
   c) Calculate the length of the radius of (C).
4) a) Calculate the coordinates of S.
   b) Use the graph to determine the solutions of the inequality $2x - 3 > -\frac{1}{2}x + 5$.
5) T is the symmetric of S with respect to K. What is the nature of SBTE? Justify the answer.
6) Find, to nearest degree, the measure of the acute angle that (Δ) makes with (x'x).

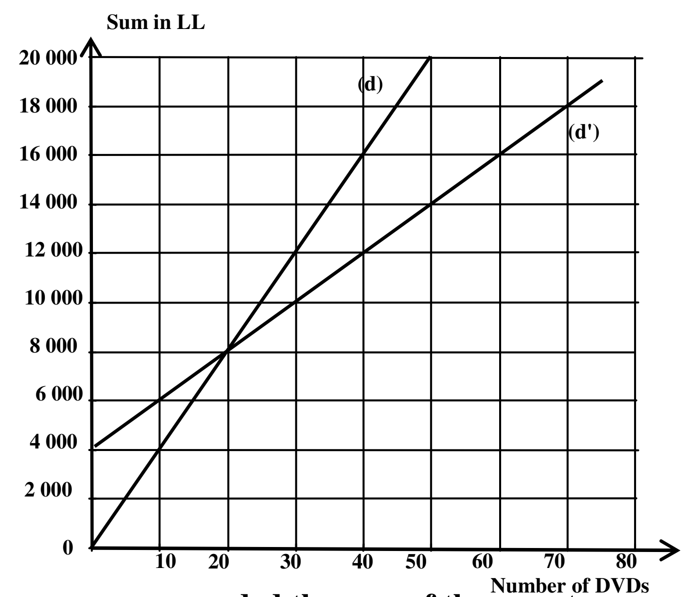
V) This problem consists of three independent parts.
A video store gives two different annual offers for renting DVDs.
offer A: pay 400 LL per each rented DVD.
offer B: pay 4000 LL as a subscription then pay 200 LL per each rented DVD.

Part 1:

1) Copy & complete this table for 15 DVDs then in terms of x DVDs:

Number of DVDs rented in a year	15	x
Sum paid using offer A in LL		
Sum paid using offer B in LL		

2) The opposite graph represents the sums A(x) & B(x) paid using offers A & B respectively.
   Answer graphically these questions:
   a) Among the lines (d) & (d'), which one represents A(x)? Why?
   b) A customer plans to rent 40 DVDs this year. Indicate which offer is the most advantageous for him. How much would he pay in this case?
   c) Copy and complete these sentences:
      i. The two offers A & B have the same price ----- LL for ------ DVDs.
      ii. From zero to 20 DVDs, offer ------ is the most advantageous for a customer.



Part 2:

In a day, 20 customers visited the store. The store manager recorded the ages of those customers and obtained the following results:

21– 33 – 18 – 46 – 21 – 21 – 33 – 46 – 30 – 15 – 15 – 18 – 18 – 46 – 33 – 30 – 21 –15 – 50 – 50.

1) Copy & complete this table:

Age in years	15	18	21	30	33	46	50
Frequency							

2) Calculate the percentage of customers whose age is over 20 years.

3) Calculate the mean age of the customers.

Part 3:

The store manager wants to get rid of his old DVD movies. There are 2646 old children's DVDs and 4410 old adults' DVDs. He decided to make identical lots of DVDs using all old DVDs.

1) What is the maximum number of lots can the manager make? Justify the answer.

2) How many children's DVDs and adults' DVDs are in each lot?

VI) Consider the triangle ABC, isosceles of vertex A & AB = 7.5 cm; BC = 12 cm.
M is the midpoint of [BC]. N is the orthogonal projection of B on (AC).

1) Draw the figure.

2) a) Express, in terms of CN, $\cos\hat{C}$ in triangle CNB.

b) Calculate $\cos\hat{C}$ using triangle CAM. Deduce that CN = 9.6 cm.

c) Calculate BN then deduce the area of triangle ABC.

3) (C) is the circumscribed circle about triangle ABN. Designate by O the center of (C).

a) Prove that O is the midpoint of [AB].

b) Prove that M is on (C).

4) (AM) cuts (BN) in P and (BA) cuts (CP) in R.

a) Show that (BR) and (CP) are perpendicular.

b) Prove that the points A, R, C and M are on same circle whose radius is to be calculated.

5) E is a variable point on (C) and F is the image of E by the translation of $\overrightarrow{OB}$.
Determine the set of points of F as E moves on (C).

Correction of Test(2)

I) 1) area=area(ABCD) - area(AEFG)= $\left(5\sqrt{21}-\sqrt{3}\right)\left(7\sqrt{3}\right)-\left(3\sqrt{2}\right)^2 = 105\sqrt{7}-39$ cm².

2) a) $A=\left(\frac{p^2+1}{2}\right)^2-\left(\frac{p^2-1}{2}\right)^2=\left(\frac{p^4+2p^2+1}{4}\right)-\left(\frac{p^4-2p^2+1}{4}\right)=\frac{4p^2}{4}=p^2$.

b) $\left(\frac{p^2+1}{2}\right)^2-\left(\frac{p^2-1}{2}\right)^2=P^2=23^2$ so P=23.

$\left(\frac{23^2+1}{2}\right)^2-\left(\frac{23^2-1}{2}\right)^2=23^2$ then $265^2-264^2=23^2$ so a=265 and b=264.

II) 1) a) $A(x)=(2x-5)^2-3(x-1)(2x-5)=(2x-5)[(2x-5)-3(x-1)]=(2x-5)(-x-2)$

b) $(2x-5)(-x-2)=0$ so $2x-5=0$ or $-x-2=0$ then $x=\frac{5}{2}$ or $x=-2$.

2) a) $B(x)=(x+3)(x+2)-(2x+4)x+(x+2)=x^2+2x+3x+6-2x^2-4x+x+2=-x^2+2x+8$.

b) $-x^2+2x+8=8$ or $-x^2+2x+8-8=0$ then $x(-x+2)=0$ then x=0 or x=2.

c) $(x+2)(4-x)=4x-x^2+8-2x=-x^2+2x+8=B(x)$.

3) a) (E) is defined when $(x+2)(4-x)\neq 0$ then $x\neq -2$ and $x\neq 4$.

b) $\frac{(2x-5)(-x-2)}{(x+2)(4-x)}=\frac{3}{2}$ or $\frac{-(2x-5)\cancel{(x+2)}}{\cancel{(x+2)}(4-x)}=\frac{3}{2}$ then $\frac{-(2x-5)}{-(x-4)}=\frac{3}{2}$ or $2(2x-5)=3(x-4)$.

c) 2(2x–5)=3(x–4) then 4x–10 = 3x–12 or x= -2, but $x\neq -2$ from part 3)a) so we have no solution.

III) Part A:

1) 40% ----------> 2688 (100 – (35+25)=40%)
100% ----------> S? so S=6720 cm^2

2) percentage $=\frac{freq}{tot}\times 100=\frac{6720}{10080}\times 100$ =66.7%. (total area= L×W=120×84=10080 cm^2).

3) a) area=area(IJKL)+ area(EFGH)- area(PQRS)= $84x+120y-xy$.

b) $x=\frac{1}{8}AB=\frac{1}{8}(120)=15$, and $84x+120y-xy=3360$ (10080 – 4480 = 3360 cm^2)
then $84(15)+120y-15y=3360$ so y=20 cm.

Part B:

$84x+120y-xy<15\%\times 10080$ so $84(12)+120y-12y<1512$ then $y<\frac{14}{3}$ so y = 1, 2, 3, or 4.

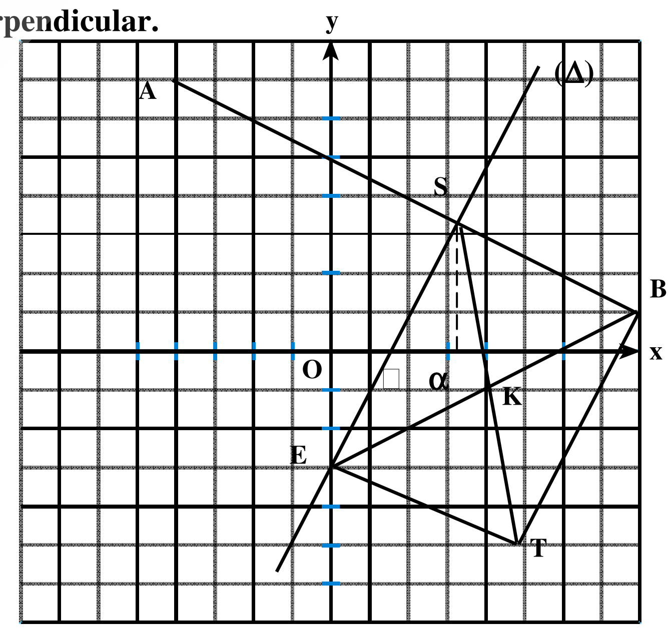
IV) 1) a) (Δ): y = 2x – 3. for x=0, y=-3; x=1,y=-1 and x=2,y=1.

b) (Δ) cuts the y-axis at E so $x_E=0$ and $y_E=2x_E-3=-3$ then E(0;-3).

2) (AB) is a general line so its equation is $y=a(x-x_A)+y_A$

$a_{(AB)}=\frac{y_B-y_A}{x_B-x_A}=-\frac{1}{2}$ then $y=-\frac{1}{2}(x+4)+7$ or (AB): $y=-\frac{1}{2}x+5$.

3) a) $a_{(AB)}\times a_{(\Delta)}=2\times -\frac{1}{2}=-1$ then (AB) & (Δ) are perpendicular.



b) The center K is the midpoint of [BE] then

$x_K=\frac{x_E+x_B}{2}=4$ & $y_K=\frac{y_E+y_B}{2}=-1$

so K(4 ;-1).

c) radius $=\frac{hyp}{2}=\frac{BE}{2}=\frac{4\sqrt{5}}{2}=2\sqrt{5}$ cm,

BE= $\sqrt{(x_E-x_B)^2+(y_E-y_B)^2}=\sqrt{80}=4\sqrt{5}$).

4) a) The coordinates of S satisfies the system

$\begin{cases} y=2x-3 \\ y=-\frac{1}{2}x+5 \end{cases}$; y=y then $x=\frac{16}{5}$ and

$y=\frac{17}{5}$ then S $\left(\frac{16}{5};\frac{17}{5}\right)$

b) The inequality $2x-3>-\frac{1}{2}x+5$ is true when

(Δ) is above (AB) then $x > \frac{16}{5}$.

5) The diagonals [BE] and [ST] bisect each other in K and $\hat{BSE} = 90^0$ so SBTE is a rectangle.

6) $\tan(\alpha)=a$ so $\tan(\alpha)=2$ then $\alpha=\tan^{-1}(2)\approx 63^o$.

V) Part 1:

1)

Number of DVDs rented in a year	15	x
Sum paid using offer A in LL	15×400=6000	400x
Sum paid using offer B in LL	4000+15×200=7000	4000+200x

2) a) (d) represents A(x) since it represents a proportional relation.

b) offer B, he would pay 12000 LL.

c) i. The two offers A & B have the same price 8000 LL for 20 DVDs at the intersection pt.

ii. From zero to 20 DVDs, offer A is the most advantageous for a customer.

Part 2:

1)

Age in years	15	18	21	30	33	46	50
Frequency	3	3	4	2	3	3	2

2) Percentage= $\frac{\text{freq.}}{\text{tot}} \times 100 = \frac{14}{20} \times 100 = 70\%$.

3) mean=29.

Part 3:

1) maximum number of lots=GCD(2646;4410)= $2\times3^2\times7^2$=882.

$2646=2\times3^3\times7^2$ and $4410=2\times3^2\times5\times7^2$

2) number of children's DVDs= $2646 \div 882=3$ and number of adults' DVDs=$4410 \div 882=5$.

VI) 1) See the figure.

2) a) $\cos\hat{C} = \frac{adj}{hyp} = \frac{CN}{12}$.

b) $\cos\hat{C} = \frac{adj}{hyp} = \frac{CM}{AC} = \frac{6}{7.5}$ =0.8. $\frac{CN}{12}$ =0.8 then CN = 9.6 cm.

c) Apply Pythagoras' theorem in right triangle BNC

$BN^2 + CN^2=BC^2$ so BN=$\sqrt{51.84}$ =7.2 cm. Area(ABC)= $\frac{BN\times AC}{2} = 27\ cm^2$.

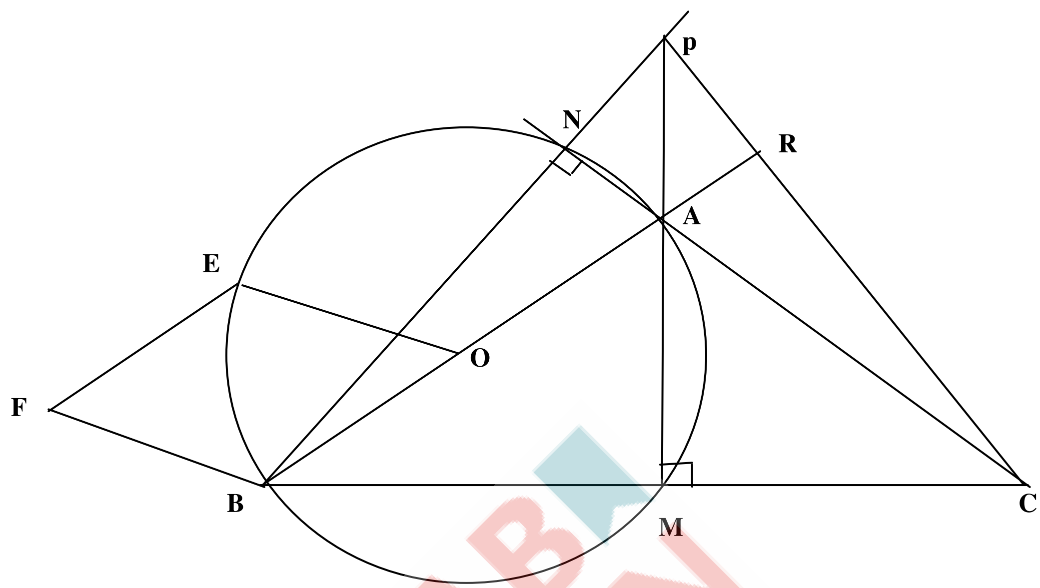
3) a) ABN is a right triangle at N so the center O of (C) is the midpoint of the hyp [AB].

b) AMB is right at M so MO=OA=OB (median relative to hyp) so MO=radius then M is on (C).

4) a) (PM) and (CN) are two heights in triangle BPC, so A is the orthocenter, then (BR) is the third height then (BR) and (CP) are perpendicular.

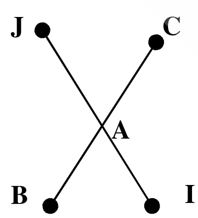
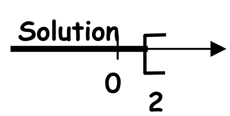
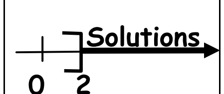
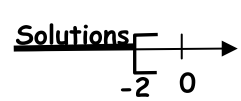
b) ARC & AMC are two right triangles of same hyp [AC], so A, R, C and M are on same circle whose radius= $\frac{AC}{2}$ =3.75 cm.

5) $\overrightarrow{OB} = \overrightarrow{EF}$ (by translation) and OE=OB=radius then OEBF is a rhombus then FB=BO=3.75 cm and B is fixed then the locus of F is the circle of center B & r=3.75 cm.



Test(3)

I) Choose, with justification, the correct answer.

	(a)	(b)	(c)
1) The volume of an advertising balloon increased by 60% under the effect of heat. To return to its original volume, volume of the balloon must reduce by ---------	40%	37.5%	60%
2) If $\frac{AB}{AC} = \frac{4}{7}$ and $\frac{AI}{AJ} = \frac{6}{10.5}$, then ... 	(BI)/ /(JC)	(BJ)/ /(IC)	none
3) $\sqrt{666^2 - 444^2 - 222^2}$ = -----	0	444	210.8
4) The graphical representation of the inequality 7x –5 < 4x +1 is ----			

II) In this question, all steps of calculations must be shown.

Consider the numbers:

$$A=\frac{35\times10^{200}+3\times10^{200}-28\times10^{200}}{10^{201}}\quad;\quad B=\frac{\sqrt{27}-\sqrt{48}+3\sqrt{75}}{7\sqrt{3}}\quad;\quad C=\frac{\frac{3}{7}-\frac{4}{5}}{\frac{7}{3}-\frac{5}{4}}$$

1) Show that A and B are integers.

2) Write C in the form of irreducible fraction.

III) Part A:

Consider the expression $F(x)=\frac{1}{4}-\left(x-\frac{7}{2}\right)^2$.

1) By using a remarkable identity, factorize F(x) and show that F(x) = (x – 3)(4 – x).

2) Develop & reduce F(x).

3) Solve the equation F(x)=0.

Part B:

Consider the rectangle (R) of perimeter P = 14 cm and area S = 12 cm².

Designate by x and y the dimensions of (R).

1) Write the perimeter of (R) in terms of x and y.

2) Deduce that x satisfies the equation x(7 – x) = 12.

3) Solve the preceding equation and deduce the dimensions of (R).

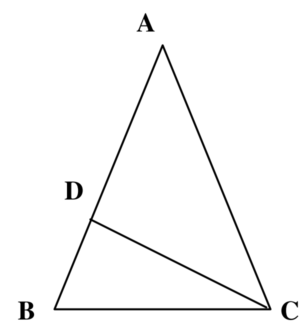
IV) The unit of measuring the angles is the degree.

ABC is an isosceles triangle of vertex A.

Suppose that $\hat{A}=x$ **and** $\hat{B}=y$.

The bisector of $\hat{C}$ **cuts [AB] in D and AD=DC.**

Establish a system of two equations in the two unknowns x and y then find x and y.

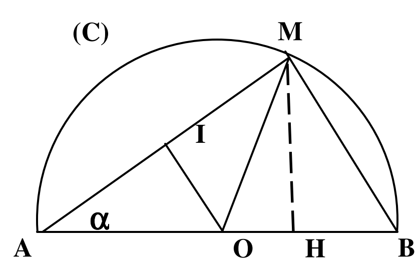


V) Consider the semi-circle (C) of center O, diameter [AB] and radius 1. M is a point on (C).

Suppose that $\widehat{MAB}=\alpha$ **where** α **is an acute angle.**

1) Justify that $\widehat{MOB}=2\alpha$.

2) I is the midpoint of [AM] and H is the foot of the perpendicular issued from M to [AB].



a) Justify that AIO is a right triangle.

b) Prove that $\cos(\alpha) = AI$ and $\cos(2\alpha) = OH$.

c) Prove that $AH = AM\times\cos(\alpha)$ then deduce that $AH = 2\cos^2(\alpha)$.

d) Show that $\cos(2\alpha) = 2\cos^2(\alpha) - 1$

3) Knowing that $\cos\left(30^{\circ}\right) = \dfrac{\sqrt{3}}{2}$, calculate the exact value of $\cos(15^{\circ})$.

VI) A person is seeking for a treasure located near two villages A and B and a castle C.
The treasure, village B and the castle C are collinear.
The treasure is at the same distance from the village A & the Village B.
On an orthonormal system of axes x'Ox; y'Oy, the villages A and B and the castle C correspond to the points A(- 2; 3), B(6; - 1) and C(8; - 7) respectively.
The unit on the plane is 1 cm and it is 120 m in reality.

Part 1:

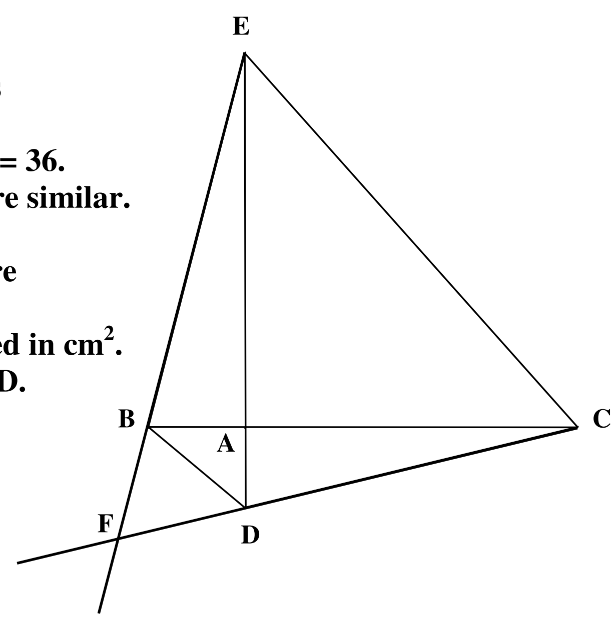
1) Place the points A, B and C.
2) Determine the slope of the line (AB).
3) Calculate the coordinates of the midpoint M of [AB].
4) Show that the equation of the perpendicular bisector, (d), of [AB] is: $y = 2x - 3$.
5) Determine the equation of the line (BC).
6) Let T be the intersection point of the lines (BC) & (d).
   Calculate the coordinates of T.

Part 2:

1) Explain why the point T represents the position of the treasure.
2) Calculate AT. Deduce the actual distance between the village A and the treasure rounded to nearest meter.

VII) *Its not required to reproduce the figure.*
The unit of length is the cm.
[BC] and [DE] are two perpendicular segments that intersect in A.
Given that AB = 8 ; AD = 6; AC = 27 ; and AE = 36.

1) Prove that the two triangles ABE and ADC are similar.
2) The lines (CD) and (EB) intersect in F.
   Prove that the two triangles DFE and BFC are similar of ratio of similarity k=1.2.
3) Denote by x the area of triangle BFD expressed in cm^2.
   a. Calculate the area of triangles BCD and EBD.
   b. Knowing that $\dfrac{\text{area(DFE)}}{\text{area(BFC)}} = k^2$,
   prove that x is the solution of the equation



$\frac{x+168}{x+105}=1.44$ then find x.

Correction of Test(3)

I) 1) rate(1)=100%+60%=1.6 and rate(1)×rate(2)=100%=1 so 1.6×rate(2)=1 or

rate(2)= $\frac{1}{1.6}$ =0.625=62.5 so it reduced by 37.5% (b)

2) If $\frac{AB}{AC}=\frac{4}{7}$ and $\frac{AI}{AJ}=\frac{6}{10.5}=\frac{4}{7}=\frac{AB}{AC}$, then (BI)/ /(JC) (converse of Thales' theorem) (a)

3) $\sqrt{666^2-444^2-222^2}=\sqrt{(111\times6)^2-(111\times4)^2-(111\times2)^2}=\sqrt{111^2\left(6^2-4^2-2^2\right)}$

$=\sqrt{111^2\times16}=111\times4=444$ (b)

4) 7x –5 < 4x +1 then 3x < 6 or x < 2 (a).

II) 1) A= $\frac{35\times10^{200}+3\times10^{200}-28\times10^{200}}{10^{201}}=\frac{(35+3-28)\times10^{200}}{10^{201}}=\frac{10\times10^{200}}{10^{201}}=\frac{10^{201}}{10^{201}}=1$

B= $\frac{\sqrt{27}-\sqrt{48}+3\sqrt{75}}{7\sqrt{3}}=\frac{3\sqrt{3}-4\sqrt{3}+15\sqrt{3}}{7\sqrt{3}}=\frac{14\sqrt{3}}{7\sqrt{3}}=2$

2) C= $\frac{\frac{3}{7}-\frac{4}{5}}{\frac{7}{3}-\frac{5}{4}}=\frac{\frac{15-28}{35}}{\frac{28-15}{12}}=\frac{\frac{-13}{35}}{\frac{13}{12}}=\frac{-13\times12}{13\times35}=\frac{-12}{35}$.

III) Part A:

1) $F(x)=\frac{1}{4}-\left(x-\frac{7}{2}\right)^2=\left[\frac{1}{2}-\left(x-\frac{7}{2}\right)\right]\left[\frac{1}{2}+\left(x-\frac{7}{2}\right)\right]=\left[\frac{1}{2}-x+\frac{7}{2}\right]\left[\frac{1}{2}+x-\frac{7}{2}\right]=(4-x)(x-3)$.

2) F(x)=(4 – x)(x – 3)= $-x^2+7x-12$.

3) (4 – x)(x – 3)=0 so x=4 or x=3.

Part B:

1) 2x + 2y = 14.

2) 2x + 2y = 14 so x + y=7 0r y=7 – x and xy=12 then x(7 – x) = 12.

3) x(7 – x) = 12 then $-x^2+7x-12=0$ so so x=4 or x=3 {part 3(A)} then l=4 cm and w=3 cm.

IV) $\hat{A}+\hat{B}+\hat{C}=180^o$ then x + y + y=180° or x + 2y=180° ----------(1)

In the isosceles triangle ADC we have $\hat{DAC}=\hat{ACD}$ then $x=\frac{y}{2}$ ----------(2)

$\begin{cases} x+2y=180 \\ x=\frac{y}{2} \end{cases}$; solving the system gives x=36° and y=72°.

V) 1) $\widehat{MOB} = \overset{\frown}{MB}$ and $\widehat{MAB} = \frac{1}{2}\overset{\frown}{MB}$ then $\overset{\frown}{MB} = 2\alpha = \widehat{MOB}$.

2) a) [AI] is the median issued from the vertex O in the isosceles triangle AOM (AO=OM=1) then (AI) is the perpendicular bisector of [AM] then AIO is a right triangle in I.

b) In the right triangle AIO, $\cos(\alpha) = \frac{AI}{AO} = \frac{AI}{1} = AI$.

In the right triangle OHM, $\cos(\widehat{MOH}) = \frac{OH}{OM}$ then $\cos(2\alpha) = \frac{OH}{1} = OH$.

c) In the right triangle AHM, $\cos(\alpha) = \frac{AH}{AM}$ then AH = AM×cos(α), but AM=2AI =2cos(α)

so $AH = 2\cos(\alpha)\times\cos(\alpha) = 2\cos^2(\alpha)$.

d) $AH = 2\cos^2(\alpha)$ then $AO+OH= 2\cos^2(\alpha)$ or $1 + \cos(2\alpha)= 2\cos^2(\alpha)$ or $\cos(2\alpha) = 2\cos^2(\alpha) - 1$.

3) $\cos(2\alpha) = 2\cos^2(\alpha) - 1$ then $\cos(2\times15) = 2\cos^2(15) - 1$ or $\cos(30)= 2\cos^2(15) - 1$

then $\cos^2(15) = \frac{\cos(30)+1}{2} = \frac{\frac{\sqrt{3}}{2}+1}{2} = \frac{\frac{\sqrt{3}+2}{2}}{2} = \frac{\sqrt{3}+2}{4}$ then $\cos(15) = \sqrt{\frac{\sqrt{3}+2}{4}}$

$= \frac{\sqrt{\sqrt{3}+2}}{2} = \frac{\sqrt{\left(\frac{\sqrt{6}+\sqrt{2}}{2}\right)^2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

VI)Part 1:

1) See the figure.

2) $a_{(AB)} = \frac{y_B - y_A}{x_B - x_A} = -\frac{1}{2}$.

3) $x_M = \frac{x_A + x_B}{2} = 2$ & $y_M = \frac{y_A + y_B}{2} = 1$ so M(2;1).

4) (d) is a general line so its equation is $y=a(x - x_M) + y_M$; $a_{(d)} \times a_{(AB)} = -1$ so a= 2.

$y= 2(x - 2) + 1$ so $y= 2x - 3$

5) (BC) is a general line so its equation is $y=a(x - x_B) + y_B$; $a_{(BC)} = \frac{y_C - y_B}{x_C - x_B} = -3$

$y= - 3(x - 6) - 1$ so $y= - 3x + 17$

6) The coordinates of T satisfies the system $\begin{cases} y = 2x - 3 \\ y = -3x + 17 \end{cases}$; y=y then x=4 and y=5 then T(4;5).

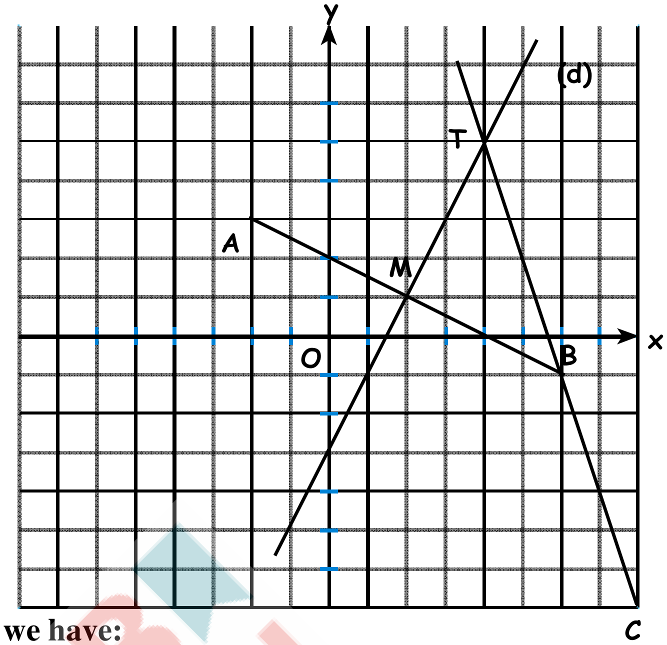
Part 2:

1) T , B and C are collinear and TA=TB (T is on the perpendicular bisector of (AB)), then T is equidistant from A & B, so T represents the position of the treasure.

2) $AT=\sqrt{(x_T - x_A)^2 + (y_T - y_A)^2} = \sqrt{40} = 2\sqrt{10}$ cm.

1 cm → 120 m

$2\sqrt{10}$ cm → x then $x \approx 759$ meter is the actual distance.



VII) 1) In the two triangles ABE and ADC we have:

$\widehat{BAE} = \widehat{CAD} = 90^0$ and $\frac{AB}{AE} = \frac{AD}{AC} = \frac{2}{9}$

then the two triangles are similar (angle and proportional sides)

2) In the two triangles DFE and BFC we have:

$\widehat{DFE} = \widehat{BFC}$ (same angle) and $\widehat{FED} = \widehat{BCF}$ (hom. angles in part 1)

then the two triangles are similar by AA.

Ratio of similarity: $\frac{\Delta DFE}{\Delta BFC} : \frac{DF}{BF} = \boxed{\frac{DE}{BC}} = \frac{FE}{FC} = \frac{42}{35} = 1.2$ so $k = 1.2$

3) a) Area(BCD)= $\frac{AD \times BC}{2} = 105\ cm^2$ and Area(EBD)= $\frac{AB \times DE}{2} = 168\ cm^2$.

b) $\frac{area(DFE)}{area(BFC)} = k^2$, then $\frac{area(BFD) + area(EBD)}{area(BFD) + area(BCD)} = 1.2^2$ so $\frac{x + 168}{x + 105} = 1.44$

or 1.44x + 151.2 = x + 168 then x=38.18.

Test(4)

I) 1) Showing all steps of calculations:

a) Calculate Q, write the answer as an irreducible fraction: $Q = \frac{1 - \frac{1}{3}}{3} \times \frac{3}{4} - \frac{1}{5}$.

b) Write R as $a\sqrt{b}$ where a & b are two integers: $R = 7\sqrt{63} - 3\sqrt{28} + \sqrt{7}$.

c) Find the positive integer S so that $\sqrt{S} = 13\sqrt{31}$

2) In this part x represents a natural number greater than 2.

a) Show that $\sqrt{(x-2)\sqrt{(x-1)(x+1)+1}+1} = x-1$.

b) Write the number $T = \sqrt{2013\sqrt{2014\times 2016+1}+1}$ in the preceding form then deduce its value.

II) Consider the expression A = (x - 2)(5 - x)

1) a) Develop and reduce A.

b) Solve the inequality $A + x^2 \leq 0$.

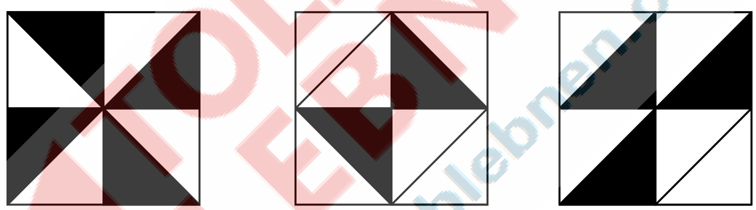
2) Consider the algebraic fraction $B = \dfrac{x+1}{2x-7} - \dfrac{x-1}{x-3}$.

a) What is the domain of definition of B?

b) Calculate B and show that $B = \dfrac{A}{(2x-7)(x-3)}$.

c) Deduce the solutions of the equation $\dfrac{x+1}{2x-7} = \dfrac{x-1}{x-3}$.

III) To make jewelry, a jeweler used triangles that have the same shape.
Some triangles are glass and the others are metal.
Three examples of jewelries are given below.
The white triangles represent the glass ones that have the same price.
The black triangles represent the metal ones that have the same price.
Designate by x the price of the glass triangle and by y that of metal triangle.



1st Jewelry 2nd Jewelry 3rd Jewelry

The price of first Jewelry is 110 000 LL & that of the second Jewelry is 91 000 LL.

1) Calculate x & y showing all steps of calculations.

2) Deduce the price of the third Jewelry.

IV) Given the inequality $-2x + \dfrac{3}{2} < 1$.

1) Solve the inequality and represent the solutions on an axis of origin O.

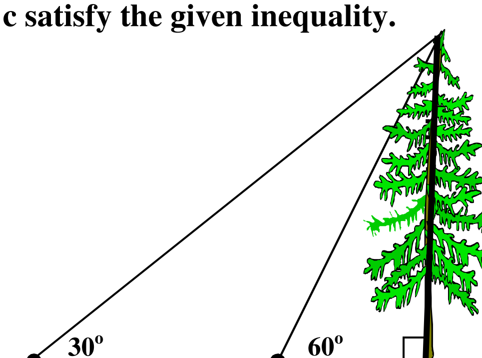
2) Consider the numbers a, b and c so that a is an integer, b is a decimal fraction and c is an irrational number.
Give a value for each number knowing that a, b, and c satisfy the given inequality.

V) ACH is a right triangle in C, B is a point of [AC].

Given that $\widehat{CAH} = a$ and $\widehat{CBH} = b$.

1) Make a figure.

2) a) Express AC in terms of CH and tan(a).



 T.Ibrahim Khalil

Express BC in terms of CH and tan(b).

b) Deduce that $AB = CH\left(\frac{1}{\tan(a)} - \frac{1}{\tan(b)}\right)$.

3) A person saw the top of a tree at an angle of 60°, then he moved back to 5 m so that he can see the tree at an angle of 30°.
Use part 2) to find the exact height of the tree.

VI) The unit of length is the cm.
On an orthonormal system of axes x'Ox, y'Oy consider the points A(5;−2), B(8; 2) and C(−3; 4).

1) Plot the points A, B, and C.
2) a) Knowing that AC=10, calculate AB and BC.
b) Prove that ABC is a right triangle in A.
c) Calculate the area of triangle ABC.
3) Determine to nearest degree the measure of $\widehat{ACB}$.
4) M is a point on [AC] where AM = x where x is a length expressed in cm.
N is on [BC] and P is on [AB] so that MNPA is a rectangle.
Denote by S(x) the area of MNPA.
a) Use Thales' theorem to express MN in terms of x.
b) Express S(x) as a function of x.
c) Verify that $S(x) = -\frac{1}{2}(x-5)^2 + \frac{25}{2}$.
d) Deduce the value of x so that $S(x) = \frac{25}{2}$ cm^2. What are the coordinates of M in this case?

VII) Consider the circle C(O;r). [AB] & [CD] are two perpendicular diameters. I is on [OC] such that $OI = \frac{1}{3}OC$ & H is the orthogonal projection of C on (AI).

1) Draw the figure.
2) Calculate AI in terms of r.
3) a) Prove that the two triangles AOI & CHI are similar.
b) Deduce that: $IH = r \times \frac{\sqrt{10}}{15}$.
4) (CH) cuts [AB] in P.
a) Prove that the points A, O, H and C are on same circle whose diameter is to be determined.
b) Prove that (PI) is perpendicular to (AC).

Correction of Test(4)

I) 1) a) $Q = \frac{1-\frac{1}{3}}{3} \times \frac{3}{4} - \frac{1}{5} = \frac{\frac{3-1}{3}}{3} \times \frac{3}{4} - \frac{1}{5} = \frac{\frac{2}{3}}{\frac{3}{1}} \times \frac{3}{4} - \frac{1}{5} = \frac{2}{9} \times \frac{3}{4} - \frac{1}{5} = \frac{1}{6} - \frac{1}{5} = \frac{5-6}{30} = \frac{-1}{30}$.

b) $R = 7\sqrt{63} - 3\sqrt{28} + \sqrt{7} = 7\times3\sqrt{7} - 3\times2\sqrt{7} + \sqrt{7} = 21\sqrt{7} - 6\sqrt{7} + \sqrt{7} = 16\sqrt{7}$.

c) $\sqrt{S} = 13\sqrt{31}$ then $\left(\sqrt{S}\right)^2 = \left(13\sqrt{31}\right)^2$ so $S = 5239$.

2) a) $\sqrt{(x-2)\sqrt{(x-1)(x+1)+1}+1} = \sqrt{(x-2)\sqrt{x^2-1+1}+1} = \sqrt{(x-2)\sqrt{x^2}+1}$

$= \sqrt{(x-2)\times x+1} = \sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = x-1$

b) $T = \sqrt{2013\sqrt{2014\times 2016+1}+1} = \sqrt{(2015-2)\sqrt{(2015-1)(2015+1)+1}+1} = 2015-1 = 2014$.

II) 1) a) $A = (x-2)(5-x) = 5x - x^2 - 10 + 2x = -x^2 + 7x - 10$.

b) $-x^2 + 7x - 10 + x^2 \le 0$ then $7x \le 10$ or $x \le \frac{10}{7}$.

2) a) $2x - 7 \neq 0$ and $x - 3 \neq 0$ so $x \neq \frac{7}{2}$ and $x \neq 3$.

b) $B = \frac{x+1}{2x-7} - \frac{x-1}{x-3} = \frac{(x+1)(x-3)-(x-1)(2x-7)}{(2x-7)(x-3)} = \frac{\left(x^2-3x+x-3\right)-\left(2x^2-7x-2x+7\right)}{(2x-7)(x-3)}$

$= \frac{-x^2+7x-10}{(2x-7)(x-3)} = \frac{A}{(2x-7)(x-3)}$.

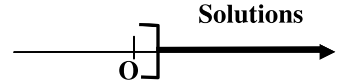
c) $\frac{x+1}{2x-7} = \frac{x-1}{x-3}$ then $\frac{x+1}{2x-7} - \frac{x-1}{x-3} = 0$ or $B=0$ that is $\frac{A}{(2x-7)(x-3)} = 0$ so $(x-2)(5-x)=0$ then $x=2$ or $x=5$.

III) 1) Using the first two jewelries: $\begin{cases} 4x+4y = 110000 \\ 6x+2y = 91000 \end{cases}$

Solving the system gives x=9000 and y=18500.

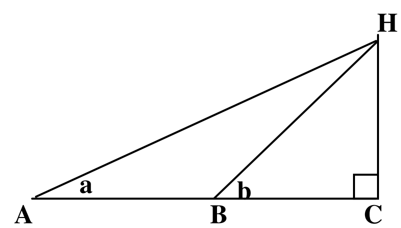
2) The price of the third Jewelry is 5x+3y=5(9000)+3(18500)=100500 LL.

IV) 1) $\frac{-2x}{1} + \frac{3}{2} < \frac{1}{1}$ so $\frac{-4x+3}{2} < \frac{2}{2}$ so $-4x+3 < 2$ or $-4x < -1$ $\times(-1)$ then $4x > 1$ or $x > \frac{1}{4}$.



2) a=1 (or 2, 3, 4, …), $b=\frac{1}{2}$ (or $\frac{3}{2}, \frac{5}{2}, \frac{3}{5}$, …), $c=\sqrt{2}$ (or $\sqrt{3}$, $\sqrt{5}$, …).

V) 1) See the figure.



2) a) In the right triangle ACH, $\tan(a) = \frac{CH}{AC}$ then $AC = \frac{CH}{\tan(a)}$.

In the right triangle BCH, $\tan(b) = \frac{CH}{BC}$ then $BC = \frac{CH}{\tan(b)}$.

b) $AB = AC - BC = \frac{CH}{\tan(a)} - \frac{CH}{\tan(b)} = CH\left(\frac{1}{\tan(a)} - \frac{1}{\tan(b)}\right)$.

3) a=30°, b=60°, AB= 5 m and height of the tree=CH.

$$AB = CH\left(\frac{1}{\tan(a)} - \frac{1}{\tan(b)}\right) \text{ then } 5 = CH\left(\frac{1}{\frac{\sqrt{3}}{3}} - \frac{1}{\sqrt{3}}\right) = CH\left(\frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) = CH\left(\frac{2}{\sqrt{3}}\right)$$

or $CH = \frac{5}{\frac{2}{\sqrt{3}}} = \frac{5\sqrt{3}}{2}$ m.

VI) 1) See the figure.

2) a) $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{25} = 5$ cm.

$BC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{125} = 5\sqrt{5}$ cm.

b) Apply the converse of Pythagoras' theorem.
$AC^2 + AB^2 = 10^2 + 5^2 = 125 = BC^2$ then ABC is a right at A.

c) Area= $\frac{AB \times AC}{2} = \frac{5 \times 10}{2} = 25$ cm^2.

3) $\tan \widehat{ACB} = \frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$ then $\widehat{ACB} = \tan^{-1}(\frac{1}{2}) \approx 27^o$.

4) a) $\frac{MN}{5} = \frac{10 - x}{10}$ then $MN = \frac{10 - x}{2}$.

b) $S(x) = MN \times MA = \frac{10 - x}{2} \times x = \frac{10x - x^2}{2}$.

c) $-\frac{1}{2}(x-5)^2 + \frac{25}{2} = -\frac{1}{2}\left(x^2 - 10x + 25\right)^2 + \frac{25}{2} = -\frac{1}{2}x^2 + 5x - \frac{25}{2} + \frac{25}{2} = -\frac{1}{2}x^2 + 5x$ =S(x).

d) $S(x) = \frac{25}{2}$ then

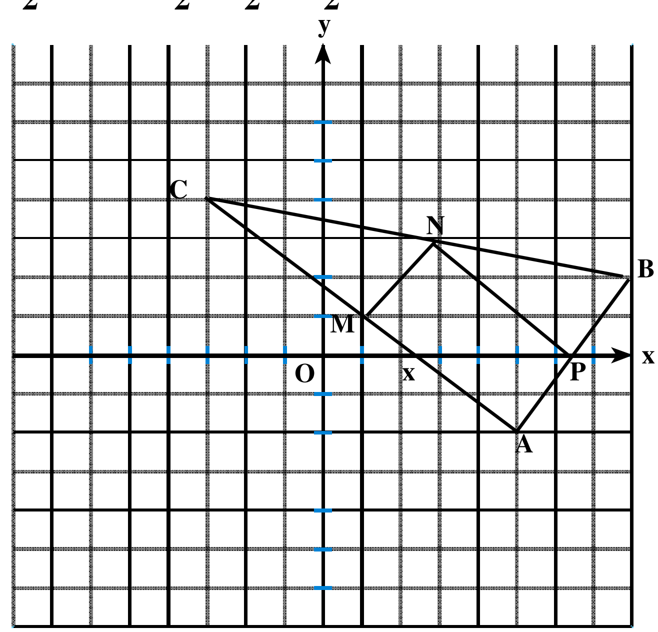
$-\frac{1}{2}(x-5)^2 + \frac{25}{2} = \frac{25}{2}$ or

$-\frac{1}{2}(x-5)^2 = 0$ then x=5 so M is

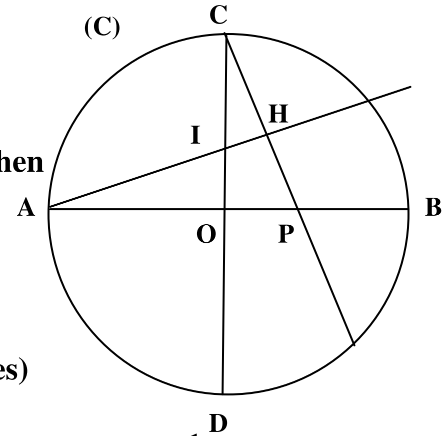
the midpoint of [AC]

$x_M = \frac{x_A + x_C}{2} = 1$ & $y_M = \frac{y_A + y_C}{2} = 1$

so M(1;1).



VII) 1) See the figure.



2) Apply of Pythagoras' theorem in the right triangle AOI.

$AO^2 + OI^2 = AI^2$ then $r^2 + \left(\frac{1}{3}r\right)^2 = AI^2$ or $r^2 + \frac{1}{9}r^2 = AI^2$ then

$AI = \sqrt{\frac{10}{9}r^2} = \frac{r}{3}\sqrt{10}$.

3) a) In the two triangles AOI & CHI we have:

$\widehat{AOI} = \widehat{CHI} = 90^o$ and $\widehat{AIO} = \widehat{CIH}$ (vertically opposite angles)

then the two triangles are similar by AA.

b) Ratio of similarity: $\frac{\Delta AOI}{\Delta CHI} : \frac{AO}{CH} = \frac{AI}{CI} = \frac{OI}{HI}$ or $\frac{AI}{CI} = \frac{OI}{HI}$ then $\frac{\frac{r}{3}\sqrt{10}}{\frac{2}{3}r} = \frac{\frac{1}{3}r}{HI}$

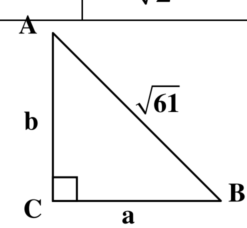
then $IH = \frac{\frac{1}{3}r \times \frac{2}{3}r}{\frac{r}{3}\sqrt{10}} = \frac{2r}{3\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = r \times \frac{\sqrt{10}}{15}$

4) a) AOC & AHC are two right triangles of same hyp [AC], so A, O, C and H are on same circle of diameter [AC].

b) (AH) and (CO) are two heights in triangle APC, so I is the orthocenter, then (PI) is the third height then (PI) and (AC) are perpendicular.

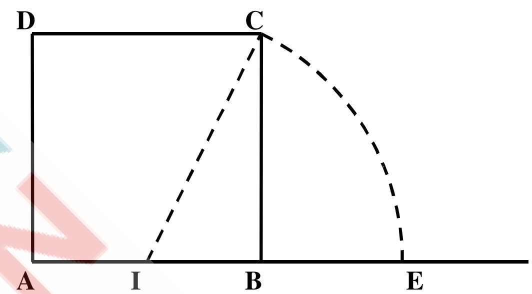
Test(5)

I) Choose, with justification, the correct answer:

1) The solutions of the equation (x – 1)(x - 2) = 6 are ---		
a. 7 & 8	**b. -1 & 4**	**c. -1 & 0**
2) How many common solutions do the two equations $(x-6)(x+1)=0$ & $x^2-3x=18$ have?		
a. 0	**b. 1**	**c. 2**
3) If $2^x = \frac{1}{\sqrt{2}}$ and $2^y = \sqrt{2}$, then 2^{y-x} =?		
a. 2	**b.** $\frac{2-\sqrt{2}}{\sqrt{2}}$	**c.** $\frac{1}{\sqrt{2}}$
4) Given a + b = 11, calculate the area of triangle ABC. 		

a. $15\ cm^2$	b. $\sqrt{15}\ cm^2$	c. $\frac{\sqrt{30}}{2}\ cm^2$
5) An item for sale is marked down by 20%. By what % must it then be marked up in order to return to the original selling price?		
a. 20 %	b. 25 %	c. 40 %

II) Consider the golden number $x = \frac{\sqrt{5}+1}{2}$.



1) Show that $x^2 = x+1$ and $\frac{1}{x} = \frac{\sqrt{5}-1}{2}$

2) ABCD is a square of side 1 cm and I is the midpoint of [AB]. The circle of center I and radius [IC] cuts [AB) in E.

a) Calculate IC.

b) Prove that the length AE is a golden number.

III) The unit of length is the cm.

ABC is a triangle where AB=8 cm, $\widehat{BAC} = 45°$ & $\widehat{ABC} = 60°$.

1) Draw the figure.

2) Calculate $\widehat{ACB}$.

3) H is the orthogonal projection of C on (AB). Prove that CH = AH.

4) Let AH=x & BH=y.

a) Establish the system of equations: $\begin{cases} x+y=8 \\ x = y\sqrt{3} \end{cases}$

b) Solve the system and show that AH= $= 12 - 4\sqrt{3}$ & $BH = 4\sqrt{3} - 4$.

IV) Given the expressions: $A(x) = 2(9x^2 - 16) - (3x+4)^2$ and $B(x) = (3x+4)(x-1)$

1) a) Factorize $9x^2 - 16$, then deduce the factorized form of A(x).

b) Solve the equation A(x) = B(x).

2) Consider the fraction $F(x) = \frac{A(x)}{B(x)}$.

a) Determine the domain of definition of F(x) then simplify F(x).

b) Evaluate $F\left(\sqrt{2}\right)$ then rationalize the answer.

c) Does the number $x = \sqrt{2}$ a solution for the inequality $F(x) \leq -9\sqrt{2}$? Justify the answer.

d) Solve the equation $F = \sqrt{3}$.

e) In this part x represents an integer greater than 1.

i) Show that $F(x) = 3 - \frac{9}{x-1}$.

ii) Deduce all values of x for which F is an integer.

V) In a school, a survey made on grade 9A and grade 9B students. Students answered the following question: "How many SMS did you send over the last 24 hours?"
This table shows the results of this survey:

Number of SMS	0	1	2	3	4	5	Total
Frequency		9		12		7	
Increasing cum. Freq.	2		24		43		

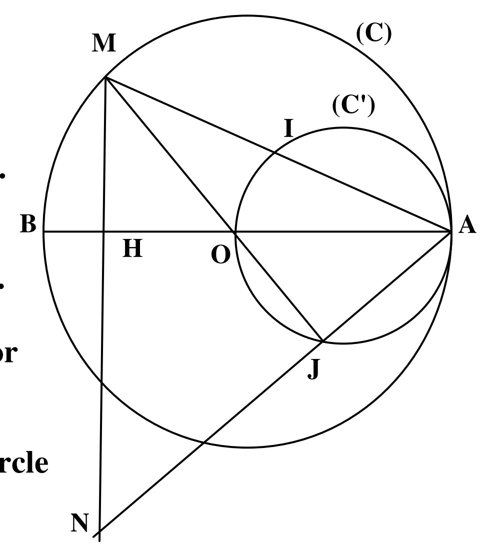
1) What is the total number of the surveyed students?
2) Find the percentage of students that sent at most 2 SMS.
3) Copy and complete the table.
4) Determine the mean.
5) Draw the increasing cum. Freq. polygon.

VI) The unit of length is the cm.

On an orthonormal system of axes x'Ox, y'Oy consider the points $A\left(-1;\frac{3}{2}\right)$ and $B\left(2;-\frac{5}{2}\right)$.

1. Plot the points A and B.
2. a) Show that the director coefficient (slope) of the line (AB) is $-\frac{4}{3}$.
   b) Write the equation of the line (AB).
3. (C) is the circle of center B and radius 5.
   a) Verify that A is on (C).
   b) (D) is the tangent line to (C) at A. Show that the equation of (D) is $y = \frac{3}{4}x + \frac{9}{4}$.
4. (D) cuts the axis of abscissa in E. Determine the coordinates of E.
5. The line (D') of equation x=1 cuts (D) in F.
   a) Plot F and determine the coordinates of F.
   b) Verify that F is the symmetric of E with respect to A.
   c) Deduce the nature of triangle BEF.

VII)
Consider the circle (C) of center O and diameter [AB].
(C') is the circle of diameter [OA].
M is a variable point on (C).
The perpendicular to (AB) passing through M cuts (AB) in H. (AM) cuts (C') again in I. (OM) cuts (C') again in J.
(MH) and (AJ) intersect in N.



1. Reproduce the figure.
2. What is the nature of triangle AIO? Justify the answer.
3. a) Prove that triangle AIO is the reduction of triangle AMB. Precise the center and calculate the scale factor of this reduction.
   b) Deduce that I is the midpoint of [AM].
4. Show that the four points N, H, O and J are on same circle whose center E is to be determined.
5. a) Show that the points N, O and I are collinear.

b) Prove that AMN is an isosceles triangle.

6. Determine the locus of F the midpoint of [MB] as M varies on (C).

Correction of Test(5)

I) 1) for x=-1, (–1 – 1)(–1 - 2) = (-2)(-3)=6 and for x=4, (4 – 1)4 - 2) = (3)(2)=6 (b)

2) (x-6)(x+1)=0 so x=6 or x=-1 and x=6 is a solution for $x^2-3x=18$ since $6^2-3(6)=18$ (b)

3) $2^{y-x}=\dfrac{2^y}{2^x}=\dfrac{\sqrt{2}}{\dfrac{1}{\sqrt{2}}}=2$ (c)

4) area=$\dfrac{a\times b}{2}$ and $(a + b)^2=a^2+b^2+2ab$ then $11^2=61+2ab$ then ab=30 ($a^2+b^2=\left(\sqrt{61}\right)^2=61$ by Pyth.)

then area = $\dfrac{30}{2}$ =15 (a)

5) rate(1) × rate(2)=100%=1 so (100% - 20%)× rate(2)=1 then rate(2)= $\dfrac{1}{0.8}$=1.25=125% that means there is a raise of 25% (b)

II) 1) $x^2=\left(\dfrac{\sqrt{5}+1}{2}\right)^2=\dfrac{5+2\sqrt{5}+1}{4}=\dfrac{6+2\sqrt{5}}{4}=\dfrac{2\left(3+\sqrt{5}\right)}{4}=\dfrac{3+\sqrt{5}}{2}$

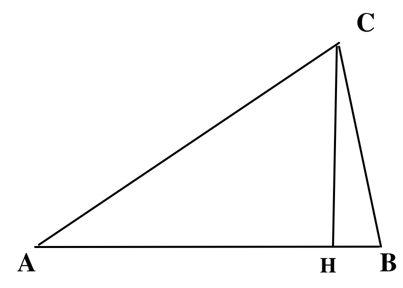
and $x+1=\dfrac{\sqrt{5}+1}{2}+1=\dfrac{\sqrt{5}+1+3}{2}=\dfrac{3+\sqrt{5}}{2}$ so $x^2=x+1$

$\dfrac{1}{x}=\dfrac{1}{\dfrac{\sqrt{5}+1}{2}}=\dfrac{2}{\sqrt{5}+1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}-1}=\dfrac{2\left(\sqrt{5}-1\right)}{5-1}=\dfrac{2\left(\sqrt{5}-1\right)}{4}=\dfrac{\sqrt{5}-1}{2}$

2) a) Applying Pythagoras' theorem in right triangle IBC gives $IC^2=IB^2+BC^2$

then $IC^2=\left(\dfrac{1}{2}\right)^2+1^2=\dfrac{5}{4}$ or IC= $\dfrac{\sqrt{5}}{2}$.

b) AE=AI+IE=AI+IC= $\dfrac{1}{2}+\dfrac{\sqrt{5}}{2}=\dfrac{\sqrt{5}+1}{2}$ is a golden number.



III) 1) See the figure.

2) $\widehat{ACB}=180-(60+45)=75^o$.

3) Triangle AHC is right isosceles in H so CH = AH.

4) a) AH+HB=AB so x+y=8 and HC=$\dfrac{BC\times\sqrt{3}}{2}$ and HB==$\dfrac{BC}{2}$ (semi-equilateral triangle HBC)

so BC=2y and HC=x=$\dfrac{2y\times\sqrt{3}}{2}=y\times\sqrt{3}$ then we have $\begin{cases}x+y=8\\ x=y\sqrt{3}\end{cases}$

b) Substitute $x = y\sqrt{3}$ in $x + y = 8$ then $y\sqrt{3} + y = 8$ or $(\sqrt{3}+1)y = 8$ or $y = \frac{8}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 4\sqrt{3} - 4$

and $x = y \times \sqrt{3} = \left(4\sqrt{3} - 4\right)\sqrt{3} == 12 - 4\sqrt{3}$ then $AH == 12 - 4\sqrt{3}$ & $BH = 4\sqrt{3} - 4$.

IV) 1) a) $9x^2 - 16 = (3x - 4)(3x + 4)$, $A(x) = 2(3x - 4)(3x + 4) - (3x + 4)^2 = (3x + 4)[2(3x - 4) - (3x + 4)]$
$= (3x + 4)(3x - 12) = 3(3x + 4)(x - 4)$.

b) $3(3x + 4)(x - 4) = (3x + 4)(x - 1)$ so $3(3x + 4)(x - 4) - (3x + 4)(x - 1) = 0$ then
$(3x+4)[3(x-4) - (x - 1)] = 0$ or $(3x+4)(2x-11)$ then $x = \frac{-4}{3}$ or $x = \frac{11}{2}$.

2) $F(x) = \frac{3(3x + 4)(x - 4)}{(3x + 4)(x - 1)}$.

a) $3x+4 \neq 0$ and $x - 1 \neq 0$ then $x \neq \frac{-4}{3}$ and $x \neq 1$. $F(x) = \frac{3\cancel{(3x+4)}(x-4)}{\cancel{(3x+4)}(x-1)} = \frac{3x-12}{x-1}$.

b) $F(\sqrt{2}) = \frac{3\sqrt{2} - 12}{\sqrt{2} - 1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{-6 - 9\sqrt{2}}{1} = -6 - 9\sqrt{2}$

c) For $x = \sqrt{2}$, $F(\sqrt{2}) = -6 - 9\sqrt{2} < -9\sqrt{2}$ then $x = \sqrt{2}$ is a solution for $F(x) \leq -9\sqrt{2}$.

d) $\frac{3x - 12}{x - 1} = \sqrt{3}$ then $3x - 12 = (x - 1)\sqrt{3}$ or $3x - 12 = x\sqrt{3} - \sqrt{3}$ then $3x - x\sqrt{3} = 12 - \sqrt{3}$

then $\left(3 - \sqrt{3}\right)x = 12 - \sqrt{3}$ that is $x = \frac{12 - \sqrt{3}}{3 - \sqrt{3}}$

e) i) $F(x) = 3 - \frac{9}{x-1} = \frac{3(x-1) - 9}{x-1} = \frac{3x-12}{x-1}$.

ii) $F(2) = 3 - \frac{9}{2-1} = -6$, $F(4) = 0$, $F(10) = 2$ so $x = 2$, 4 and 10.

V) 1) $43+7=50$ is the total number of the surveyed students.

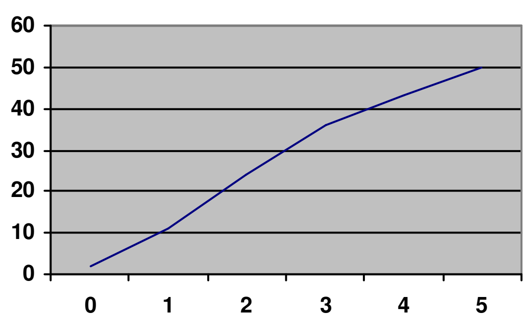
2) percentage $= \frac{freq}{tot} \times 100 = \frac{24}{50} \times 100 = 48\%$.

3)

Number of SMS	0	1	2	3	4	5	Total
Frequency	2	9	13	12	7	7	50
Increasing cum. Freq.	2	11	24	36	43	50	

4) $\text{mean} = \frac{0 \times 2 + 1 \times 9 + \cdots}{50} = 2.68$.

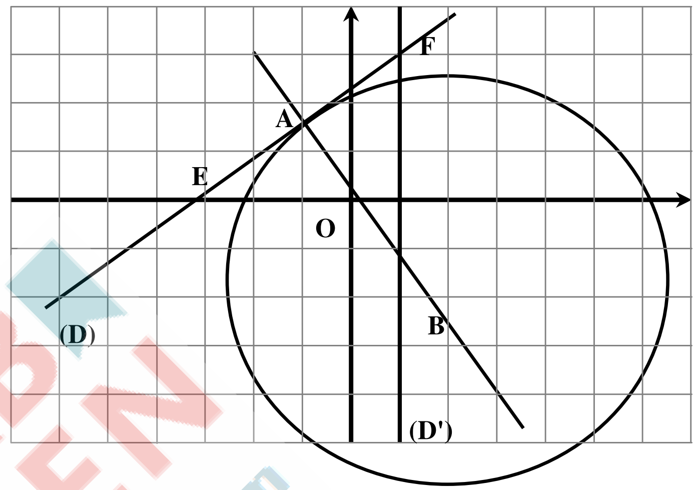
5)



VI) 1. Figure.

2\. a) $a_{(AB)}\ \dfrac{y_B - y_A}{x_B - x_A} = -\dfrac{4}{3}$.

b) Equation of the line (AB) is $y = a(x - x_A) + y_A = -\dfrac{4}{3}x + \dfrac{1}{6}$.



3\. a) $AB = \sqrt{25} = 5$ = radius then A is on (C).

b) Equation of the line (D) is $y = a(x - x_A) + y_A$ and $a \times a' = -1$ so $a = \dfrac{3}{4}$ then $y = \dfrac{3}{4}x + \dfrac{9}{4}$.

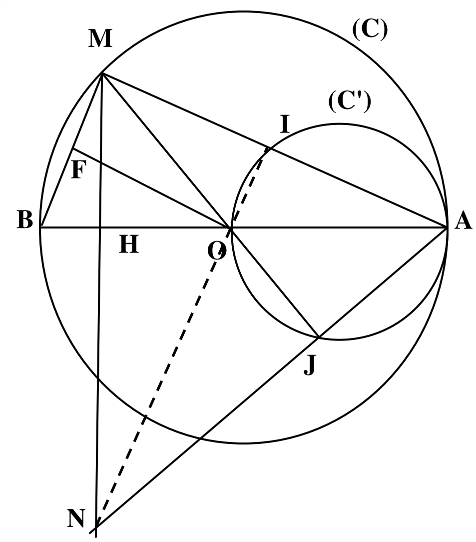
4\. The coordinates of $E\left(\dfrac{-b}{a};0\right)$ so E(-3;0).

5\. a) (D')//(y'y) that cuts (D) in F.
The coordinates of F satisfies the system $\begin{cases} y = \dfrac{3}{4}x + \dfrac{9}{4} \\ x = 1 \end{cases}$ then F(1;3)

b) We prove that $x_A = \dfrac{x_E + x_F}{2}$ and $y_A = \dfrac{y_E + y_F}{2}$ that is A is the midpoint of [EF].

c) (AB) is the perpendicular bisector of [EF].

VII) 1. Figure.



2\. AIO is right at I (inscribed in semi-circle).

3\. a) $\widehat{AIO} = \widehat{AMB} = 90^o$ and $\widehat{IAO} = \widehat{BAM}$ (same angle)
So triangles are similar of ratio $\dfrac{AO}{AB} = \dfrac{1}{2}$ & center A.

b) $\dfrac{AI}{AM} = \dfrac{1}{2}$ so $AI = \dfrac{1}{2}AM$ then I is the midpoint of [AM].

4\. NHO and J ON are two right triangles of common hyp. [ON] then N, H, O and J are on same circle of center E the midpoint of [ON].

5\. a) O is the orthocenter in triangle AMN, then (NO) is the third height or (ON) is perpendicular to [AM], but (OI) is the perpendicular bisector of [AM] in the isosceles triangle AOM, then N, O and I are collinear.

b) (NI) is the perpendicular bisector of [AM].

6\. OFB is right triangle at F of fixed hyp. [OB], so the locus of F is the circle of diameter [OB].

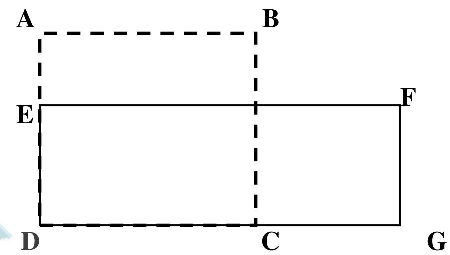
Test(6)

I) 1) Given the two numbers a & b where $a^2+b^2=8$ & $a+b=2\sqrt{3}$.

a) Show that ab = 2.

b) Expand $\left(a^2+b^2\right)^2$. Deduce the value of a^4+b^4.

2) The opposite figure composed of a square ABCD and a rectangle DEFG.
E is a point of the segment [AD].
C is a point of the segment [DG].
The length AB may vary but we always have:
AE = 15 cm and CG= 25 cm.



a) In this question, assume that AB = 40 cm.

i. Calculate the area of ABCD.

ii. Calculate the area of DEFG.

b) In this question, assume that AB = x cm.
Determine the length AB so that the area of ABCD is equal to the area of DEFG.

II) Show that:

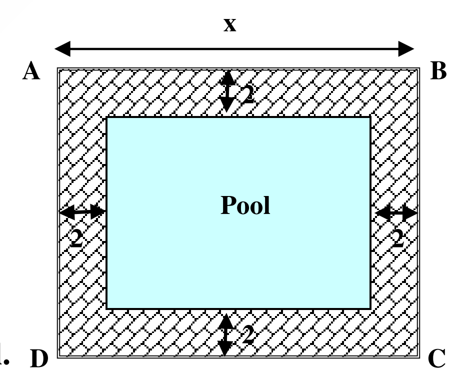
1) $\dfrac{\cos x-2\cos^3 x}{2\sin^3 x-\sin x}=\dfrac{1}{\tan x}$ where x is an acute angle.

2) $\left(1+\tan^2\alpha\right)\left(1-\sin^2\alpha\right)=1$ where α is an acute angle.

III) Jad wants to build a rectangular swimming pool in his garden according to the following conditions:

- The pool must surrounded by an area covered with slabs of width 2 m;
- The total surface (pool and slabs) is a rectangle of area 300 m^2.

ABCD is the rectangle representing the total surface.
Suppose that AB=x where x is a length expressed in m and $4 \le x \le 75$ and designate by S(x) the area of the swimming pool.

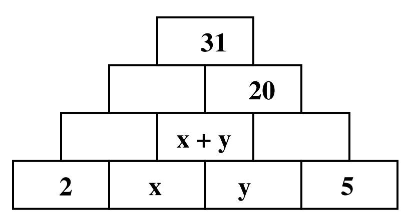


1) Express AD in terms of x.

2) Show that $S(x)=316-4x-\dfrac{1200}{x}$.

3) A friend of Jad assured him that the pool has a maximum area if it is square.
Assuming that this statement is true, calculate x then deduce that the area of the pool is $316-80\sqrt{3}$ m^2.

IV) In the opposite pyramid, the number in a brick is equal to the sum of two numbers in the two bricks that support it.
Determine x and y.



V) ABCD is a rectangle of center O. E & F are two points where
$\overrightarrow{AE}=\overrightarrow{BD}$ & $\overrightarrow{AF}=\overrightarrow{AC}+\overrightarrow{AB}$.

1) Draw the figure.
2) Show that $\overrightarrow{ED} = \overrightarrow{CF}$.
3) Denote by t the translation that transforms A to B.
 a) Draw O' the image of O by the translation t.
 b) Show that $\overrightarrow{BO'} = \frac{1}{2}\overrightarrow{BF}$. What do you conclude?
 c) (C) is the circle of center D & passing through C.
 Determine the center & radius of the circle (C'), the image of (C) by the translation t.
4) a) Complete by a vector:

$\overrightarrow{AO} + \overrightarrow{AB} = ----$; $\overrightarrow{AE} + \overrightarrow{DC} = ----$; $\overrightarrow{DO} + --- = \vec{0}$

 b) Complete by a number:

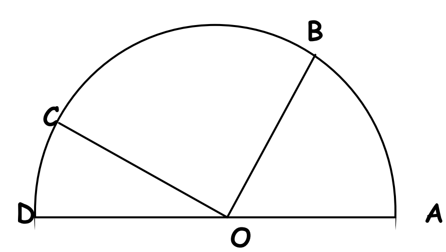
$\overrightarrow{CD} = ---\overrightarrow{FE}$; $\overrightarrow{OO'} = ---\overrightarrow{DF}$

VI) On an orthonormal system of axes, consider the points $A\left(-1;\frac{7}{2}\right)$, $B\left(5;\frac{9}{2}\right)$ and $C\left(4;-\frac{3}{2}\right)$.

1) Draw the figure.
2) Calculate the coordinates of M the midpoint of [AC].
3) a) Knowing that AB= $\sqrt{37}$, calculate the lengths AC and BC.
 b) Deduce that ABC is an isosceles triangle but not right.
 c) What conclusion can you draw concerning the lines (AC) and (BM)?
4) The circle (C) of center M and radius MB cuts (BM) again in D.
 Determine the coordinates of D. What is the nature of ABCD? Justify the answer.
5) $E\left(\frac{7\sqrt{2}+3}{2};1\right)$ is a point on (C).

 Construct E then determine the equation of (d) the tangent to (C) at E.
6) G is the image of M by the translation of $\overrightarrow{MA} + \overrightarrow{MD}$. Determine the coordinates of G.
7) (D) is an increasing line passing through G and makes angle 60^0 with (x'x).
 a) Draw (D).
 b) The equation of (D) is in the form y=ax + b.
  i) Define a & b.
  ii) Calculate a & b.
 c) Write the equation of (D') the image of (D) by the translation of $\overrightarrow{GO}$.

VII) The unit of length is the cm.
 Consider the semicircle (C) of radius 4, center O and diameter [AD]. B and C are two points on (C) where AB = 4 and BOC is a right triangle in O.
 I is the intersection point of (BD) and (CA) and (BD) cuts (OC) in M.



1. Reproduce and complete the figure.

2. Calculate BC and BD.
3. a) Prove that the two triangles BIC and AID are similar.

b) Deduce that the ratio of the perimeter of triangle BIC to that of triangle AID is $\frac{\sqrt{2}}{2}$.

4. a) Calculate the measure of $\widehat{ADB}$.
   b) Prove that the two triangles ABD and MOB are similar.
   c) Deduce the value of OMxBD then calculate OM.
5. (DC) and (AB) intersect at E. Prove that (EI) is perpendicular to (AD).

Correction of Test(6)

I) 1) a) $(a + b)^2 = a^2 + b^2 + 2ab$ then $\left(2\sqrt{3}\right)^2 = 8 + 2ab$ so $12 - 8 = 2ab$ then ab=2.

b) $\left(a^2 + b^2\right)^2 = a^4 + b^4 + 2a^2b^2$ then $8^2 = a^4 + b^4 + 2(2)^2$ then $a^4 + b^4 = 64 - 8 = 56$.

2) a) i)Area(ABCD)= $S^2 = 40^2 = 1600$ cm^2.
ii) Area(DEFG)= L×W=65×25=1625 cm^2.
b) Area(ABCD)=Area(DEFG) so $x^2 = (x + 25)(x - 15)$ then $x^2 = x^2 + 10x - 375$ or x =37.5 cm^2.

II) 1) $\frac{\cos x - 2\cos^3 x}{2\sin^3 x - \sin x} = \frac{\cos x\left(1 - 2\cos^2 x\right)}{\sin x\left(2\sin^2 x - 1\right)} = \frac{\cos x\left[1 - 2(1 - \sin^2 x)\right]}{\sin x\left(2\sin^2 x - 1\right)} = \frac{\cos x\left[1 - 2 + 2\sin^2 x\right]}{\sin x\left(2\sin^2 x - 1\right)}$

$= \frac{\cos x\left(2\sin^2 x - 1\right)}{\sin x\left(2\sin^2 x - 1\right)} = \frac{\cos x}{\sin x}$ and $\frac{1}{\tan x} = \frac{1}{\frac{\sin x}{\cos x}} = \frac{\cos x}{\sin x}$

2) $\left(1 + \tan^2\alpha\right)\left(1 - \sin^2\alpha\right) = \left(1 + \frac{\sin^2\alpha}{\cos^2\alpha}\right)\left(1 - \sin^2\alpha\right) = \left(\frac{1}{\cos^2\alpha}\right)\left(\cos^2\alpha\right) = 1$.

III) 1) area=AB×AD then 300=x×AD or AD=$\frac{300}{x}$

2) S(x)=length×width=$(x - 4)(\frac{300}{x} - 4) = 300 - 4x - \frac{1200}{x} + 16 = 316 - 4x - \frac{1200}{x}$.

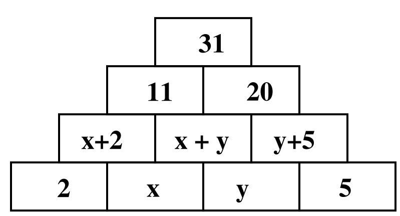
3) pool is a square when length=width then $x - 4 = \frac{300}{x} - 4$ then $x - \frac{300}{x} = 0$

then $\frac{x^2 - 300}{x} = 0$ so $x^2 - 300 = 0$ then $x = \sqrt{300} = 10\sqrt{3}$ m.

Area=side^2 = $(x - 4)^2 = \left(10\sqrt{3} - 4\right)^2 = 316 - 80\sqrt{3}$ m^2.

IV) x+2+x+y=11 and x+y+y+5=20 then $\begin{cases} 2x + y = 9 \\ x + 2y = 15 \end{cases}$

Solving the system gives x=1 and y=7.



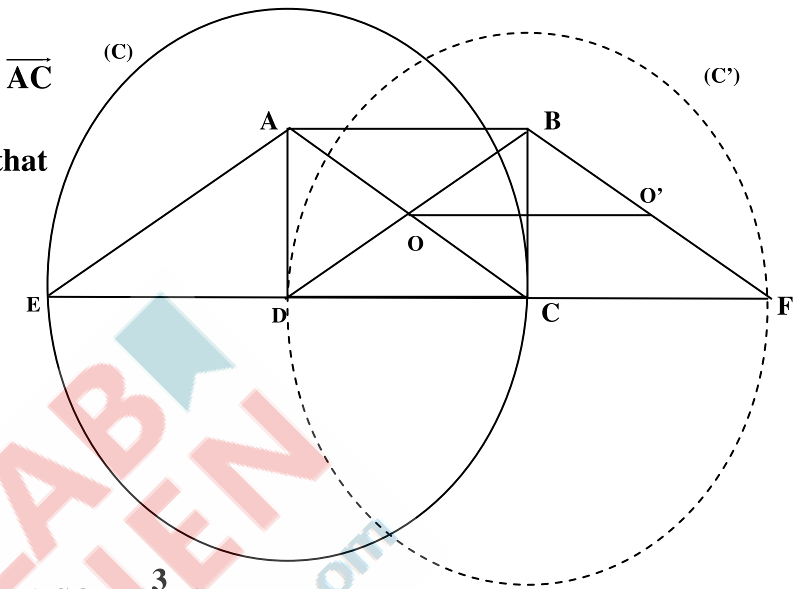
V) 1) See the figure.

2) ABDE is a parm since $\overrightarrow{AE}=\overrightarrow{BD}$ so $\overrightarrow{ED}=\overrightarrow{AB}$ and $\overrightarrow{CF}=\overrightarrow{AB}$ (by translation) then $\overrightarrow{ED}=\overrightarrow{CF}=\overrightarrow{AB}$.

3) a) figure.

b) ABO'O is a parm since $\overrightarrow{OO'}=\overrightarrow{AB}$ so $\overrightarrow{AO}=\overrightarrow{BO'}$ and $\overrightarrow{AC}=\overrightarrow{BF}$, but $\overrightarrow{AO}=\frac{1}{2}\overrightarrow{AC}$ then $\overrightarrow{BO'}=\overrightarrow{AO}=\frac{1}{2}\overrightarrow{BF}$. We conclude that O' is the midpoint of [BF].

c) Center is C the image of D by the translation t and radius CF=CD.



4) a) $\overrightarrow{AO}+\overrightarrow{AB}=\overrightarrow{AO'}$; $\overrightarrow{AE}+\overrightarrow{DC}=\overrightarrow{AD}$

$\overrightarrow{DO}+\overrightarrow{BO}=\vec{0}$

b) $\overrightarrow{FE}=3\overrightarrow{CD}$; $\overrightarrow{OO'}=\frac{1}{2}\overrightarrow{DF}$

VI) 1) Seethe figure.

2) $x_M=\frac{x_A+x_C}{2}=\frac{3}{2}$ and $y_M=\frac{y_A+y_C}{2}=1$ SO $M(\frac{3}{2};1)$

3) a) AB=$\sqrt{37}$, AC=$\sqrt{(x_C-x_A)^2+(y_C-y_A)^2}=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$ and BC=$\sqrt{37}$.

b) AB=BC=$\sqrt{37}$ and $AB^2+BC^2=37+37=74\neq AC^2$ then ABC is an isosceles triangle but not right.

c) (AC) and (BM) are perpendicular since the median (AM) is the perpendicular bisector of [AC] in the isosceles triangle ABC.

4) M is the midpoint of the diameter [BD] then

$x_M=\frac{x_B+x_D}{2}=\frac{5+x_D}{2}=\frac{3}{2}$ then $x_D=-2$ and $y_M=\frac{y_B+y_D}{2}=\frac{\frac{9}{2}+y_D}{2}=1$ then $y_D=-\frac{5}{2}$

ABCD is a rhombus since diagonals [BD] and [AC] are perpendicular bisectors to each others.

5) E is the intersection between the line y=1(parallel to (x'x)) and (C).

(d) is perpendicularto the radius (ME) at E and (ME)// (x'x) so (d) // (y'y) then its equation is $x=x_E=\frac{7\sqrt{2}+3}{2}$.

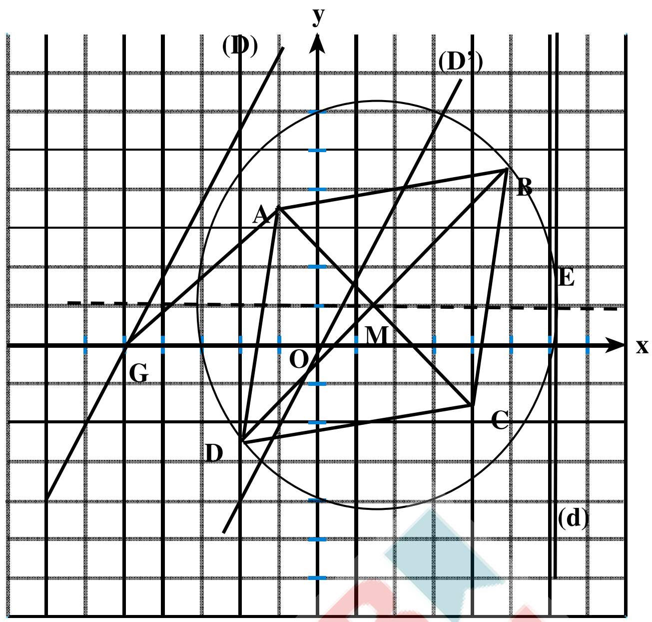
6) $\overrightarrow{AG}=\overrightarrow{MD}$ so $x_G-x_A=x_D-x_M$ then $x_G=-5$ and $y_G-y_A=y_D-y_M$ then $y_G=0$ so G(-5;0).

7) a) Figure.

b) i) a is the slope of (D) &b is the y-intercept of (D).

ii) $a=\tan(60^0)=\sqrt{3}$ then $y=\sqrt{3}x+b$ and $y_G=\sqrt{3}x_G+b$ so $0=-5\sqrt{3}+b$ or $b=5\sqrt{3}$.

c) (D)// (D') (by translation) and passing through O so its equation is $y=ax=\sqrt{3}x$.



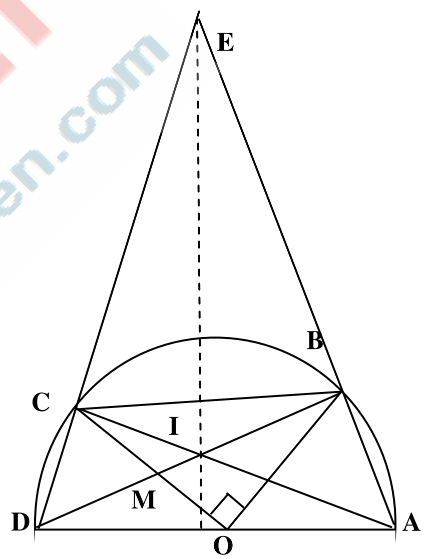
VII) 1. See the figure.

2. By Pythagoras theorem in right triangle BOC,

$BC^2=OC^2+OB^2=16+16=32$ **so** $BC=\sqrt{32}=4\sqrt{2}$ **cm.**

By Pythagoras theorem in right triangle ABD,

$AB^2+BD^2=AD^2$ **so** $16+BD^2=64$ **so** $BD=\sqrt{48}=4\sqrt{3}$ **cm.**



3. a) $\widehat{BIC}=\widehat{AID}$ **(vertical angles) and** $\widehat{IBC}=\widehat{IAD}=\frac{1}{2}\overset{\frown}{CD}$

so BIC and AID are similar by AA.

b) Ratio of perimeters= $\frac{IB+IC+BC}{IA+ID+AB}$ **but**

ratio of similarity= $\frac{IC}{ID}=\frac{BI}{AI}=\frac{BC}{AD}=\frac{4\sqrt{2}}{8}=\frac{\sqrt{2}}{2}$

then $\frac{\frac{\sqrt{2}}{2}\cancel{(IA+ID+AB)}}{\cancel{IA+ID+AB}}=\frac{\sqrt{2}}{2}$.

4. a) Triangle AOB is an equilateral triangle since OB=OA=AB=4 cm

then ABD is a semi-equilateral triangle(90^o; 60^o; 30^o**) so** $\widehat{ADB}=30^o$.

b) $\widehat{ABD}=\widehat{MOB}=90^o$ **and** $\widehat{ADB}=\widehat{OBM}$ **(base angles of isosceles triangle BOD)**

so the two triangles ABD and MOB are similar by AA.

c) ratio of similarity= $\frac{AB}{OM}=\frac{AD}{MB}=\frac{BD}{OB}$ **or** $\frac{AB}{OM}=\frac{BD}{OB}$ **then** $OM\times BD=AB\times OB=$

$8\text{x}4=32$ **then** $OM=\frac{32}{4\sqrt{3}}=\frac{8}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{8\sqrt{3}}{3}$ **cm.**

5. (AC) and (BD) are two heights in triangle AED then I is the orthocenter so (EI) is the third heights in triangle AED then (EI) is perpendicular to (AD).